Estimation

Assms:

$$E(u) = 0$$

$$E(x_1 u) = 0$$

$$E(x_2 u) = 0$$

OLS estimates: $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$

of β_0, β_1, and β_2

$$\frac{1}{n} \sum_{i=1}^{n} \boxed{\left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \hat{\beta}_2 x_{i2}\right)} = 0$$

$$\frac{1}{n} \sum_{i=1}^{n} x_{i1} \square = 0$$

$$\frac{1}{n} \sum_{i=1}^{n} x_{i2} \square = 0$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} \quad \text{(fitted value)}$$

$$\hat{u}_i = y_i - \hat{y}_i$$

Goodness of fit: $R^2 = SSE / SST$

$= 1 - \frac{SSR}{SST}$ non↓ing in k (# of x's)

Adjusted R^2: $\bar{R}^2 = 1 - \frac{\frac{SSR}{(n-k-1)}}{\frac{SST}{(n-1)}}$

As k ↑ SSR ↓ $(n-k-1)$ ↓

Expected Values

unbiased OLS estimates $E(\hat{\beta}_j) = \beta_j$ $j = 0, 1, \ldots, k$ (# x's)

under certain ass^n^s

- linear in parameters (i.e. β's)
- random sampling
- variation in each x
- u unrelated to each x

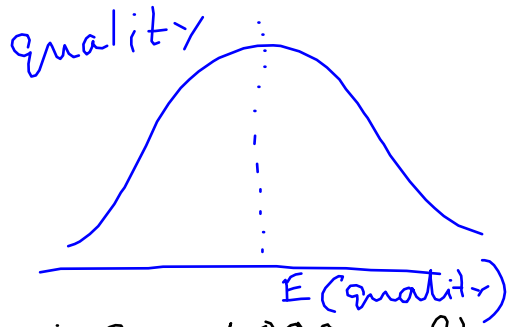
No linear rel^n^ship among x's

$E(u | x_1, x_2, \ldots x_k) = E(u) = 0$

$wage = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 age + \ldots + u$

$age = 6 + educ + exper$

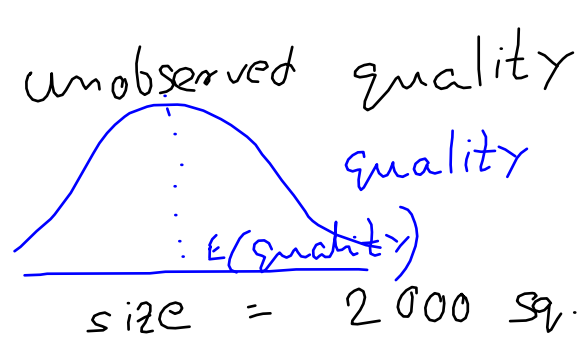
$$\text{property value} = \beta_0 + \beta_1 \text{size} + \beta_2 \#\text{BRs} + \beta_3 \text{whether garage} + \beta_4 \text{lot size} + u$$



size = 1000 sqft.

2 BRs
with garage

lot size 2000 sqft.



size = 2000 sq.ft.

3 BRs

no garage

lot size = 2500 sq.ft.

Omitted Variable Bias

True model satisfying cond^{n}s for unbiasedness:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

wage — educ — IQ
bwght — smoking — alcohol

x_2 : omitted; estimate:

$$y = \beta_0 + \beta_1 x_1 + v$$

obtain: $\tilde{\beta}_0$ and $\tilde{\beta}_1$ → biased

$$E(\tilde{\beta}_j) \neq \beta_j \qquad j = 0, 1$$

Bias depends on β_2 and corr. b/w x_2 (omitted) and x_1 (included)