Single Hypothesis – Single Parameter

Null : $H_0 : \beta_j = a_j$ $a_j = 0 \rightarrow$ special case

Alternative : $H_1 : \beta_j \neq a_j \rightarrow$ two tailed test

$$\boxed{\begin{array}{l} > \quad \text{would be} \\ < \quad \text{one-tailed test} \end{array}}$$

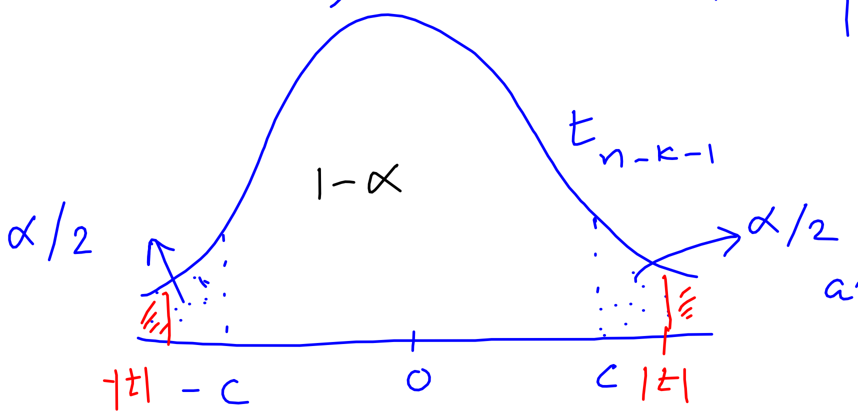
Test statistic :

t "

t ratio $t_{test} = \dfrac{\hat{\beta}_j - a_j}{se(\hat{\beta}_j)}$

If H_0 is true, unlikely $|t_{test}| > c$

↳ critical value



area $\boxed{\,\vdots\,} = \alpha$

Rejection rule: reject H_0 if $|t_{test}| > c$ else fail to reject H_0

α = significance level (or size)
$= P(\text{rej. } H_0 \mid H_0 \text{ true})$

Equivalent rejection rule:

reject H_0 if a_j is beyond $c \cdot se(\hat{\beta}_j)$ from $\hat{\beta}_j$.

$$\frac{\hat{\beta} - a}{se} > c \qquad \Bigg| \qquad \frac{\hat{\beta} - a}{se} < -c$$

$$\hat{\beta} - a > c \cdot se \qquad \Bigg| \qquad \hat{\beta} - a < -c \cdot se$$

$$\hat{\beta} - c \cdot se > a \qquad \Bigg| \qquad \hat{\beta} + c \cdot se < a$$

Fail to reject H_0 if a_j is within

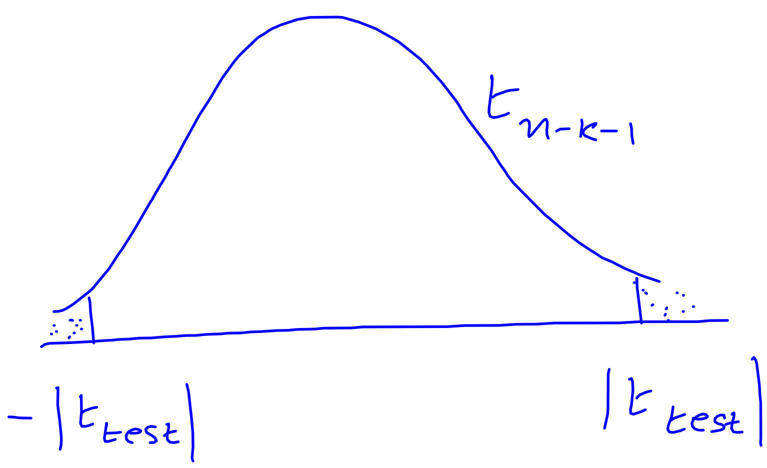
$$\left[\hat{\beta}_j - c \cdot se(\hat{\beta}_j),\ \hat{\beta}_j + c \cdot se(\hat{\beta}_j)\right]$$

$(1-\alpha)$ confidence interval for β_j: $\hat{\beta}_j \pm c \cdot se(\hat{\beta}_j)$

↳ confidence level $P(\text{not rej. } H_0 \mid H_0 \text{ true})$

Another equivalent rejection rule:

reject H_0 if area beyond $|t_{test}|$ & $-|t_{test}|$ $< \alpha$



area $\boxed{\ }$ $= 2P(t > |t_{test}|)$

$\downarrow$

p-value

rej. H_0 if p-value $< \alpha$

nbasal :

$wage = \beta_0 + \beta_1 \text{ points} + \beta_2 \text{ rebounds} + \beta_3 \text{ assists} + u$

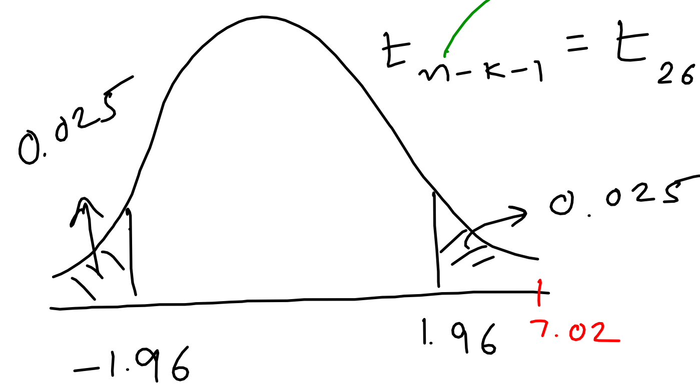
$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

$t_{test} = \dfrac{\hat{\beta}_1 - 0}{se(\hat{\beta}_1)} = 7.02$

$n = 269$
$k = 3$

$t_{n-k-1} = t_{265}$



$\alpha = 0.05$

c for t_{n-k-1}

Table G2 : t

" G1 : $N(0,1)$

$\boxed{\text{Rej. } H_0 \because t_{test} > 1.96}$

$H_0 : \beta_1 = 70$

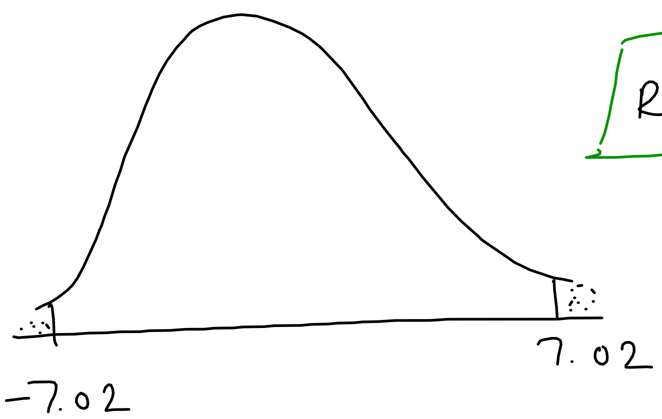
$H_1 : \beta_1 \neq 70$

0.95 CI : $\hat{\beta}_1 \pm c \cdot se(\hat{\beta}_1)$

81.194   1.96   11.569

[58.41 , 103.97]

$\boxed{\text{Rej. } H_0 \because \text{CI excludes } 0}$



$\boxed{\text{Rej. } H_0 \because \text{p-value} < 0.05}$

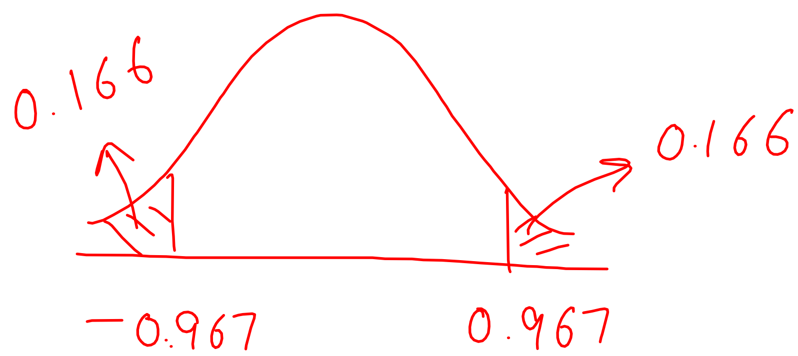
practically 0

$$t_{test} = \frac{\hat{\beta}_1 - 70}{se(\hat{\beta}_1)} = 0.967$$

$\alpha = 0.05$

$H_0 : \beta_1 = 70$

$H_1 : \beta_1 \neq 70$



Fail to rej. H_0.

$$\text{p-value} = 2 \times 0.166$$
$$= 0.312$$

Single Hypothesis - Multiple Parameters

$$H_0 : \beta_j = \beta_l$$

$$H_1 : \beta_j \neq \beta_l$$

$$t_{test} = \frac{\hat{\beta}_j - \hat{\beta}_l}{se(\hat{\beta}_j - \hat{\beta}_l)}$$

nbasal :

$$H_0 : \beta_1 = \beta_2$$

$$H_1 : \beta_1 \neq \beta_2$$

Multiple Hypotheses

$H_0 : \beta_j = 0 , \beta_\ell = 0$

$H_1 :$ at least β_j or $\beta_\ell \neq 0$

unrestricted model : H_0 not imposed

restricted " : H_0 imposed

(by omitting x_j and x_ℓ)

Test statistic based on comparing fit across the 2 models.

Follows F distribution.

SSR_r : SSR in restricted

$$F_{test} = \frac{\left(SSR_r - SSR_{ur}\right)/q}{SSR_{ur}/(n-k-1)}$$

SSR in unrestricted

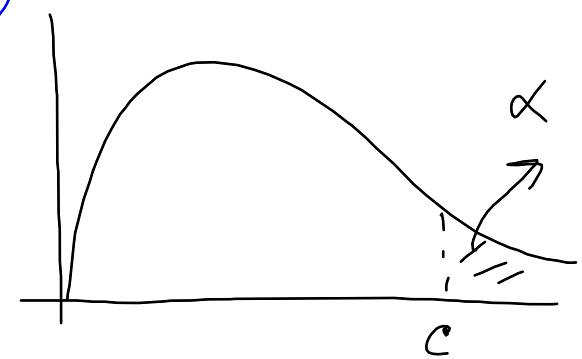
$F_{test} \sim F_{q,\, n-k-1}$

q : numerator df (# β's tested)

$n-k-1$: denominator df

sample size

x's (unrestricted)



Reject H_0 if $F_{test} > c$

Critical values : Tables G.3a, G.3b, & G.3c

$q = 1$ $\qquad F_{1,\, n-k-1} = t^2_{n-k-1}$

Also, $$F_{test} = \frac{\left(R^2_{ur} - R^2_{r}\right)/q}{\left(1 - R^2_{ur}\right)/(n-k-1)}$$

R^2 restricted

R^2 unrestricted

nbasal : $H_0 : \beta_1 = 0 , \beta_2 = 0$

H_1 : not H_0

$$F_{test} = 93.52$$

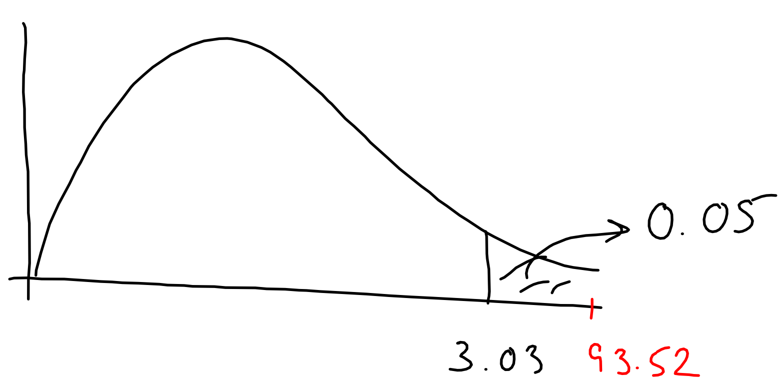
3 (from table)

c for $\alpha = 0.05$, $F_{2,265} = 3.03$

[redacted]

[redacted]

Reject H_0.



Special case: overall significance of regression

$H_0: \beta_1, \ldots, \beta_k = 0$

H_1: at least one of $\beta_1 \ldots \beta_k \neq 0$

R^2_{ur} : usual R^2

R^2_r : 0

$q = k$

$$F_{test} = \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

nbasal: $H_0: \beta_1, \beta_2, \beta_3 = 0$

H_1: not H_0

$F_{test} = 80.07$

c for $\alpha = 0.05$ $F_{3,265} = 2.60$ (table)

[redacted]

Reject H_0.