Ch: 4 (Inference) cont.

Sampling Distributions

$\hat{\beta}_j = \beta_j +$ a term involving u

$\downarrow$

its distribution follows from that of u

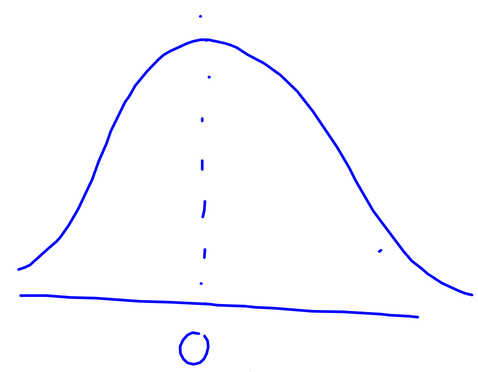
Ass^n $\quad u \rightarrow$ indep. of all x's

$\hookrightarrow$ normal $\qquad E(u) = 0$

$\qquad\qquad\qquad\qquad var(u) = \sigma^2$

$u \sim \overset{\text{normal}}{N}(\underset{\text{avg.}}{0}, \underset{\text{var.}}{\sigma^2})$

$\sim$: is distributed as



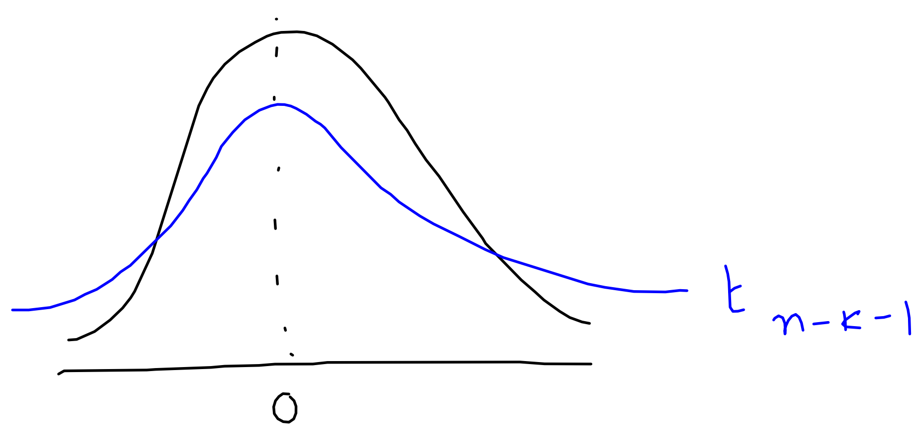
Given all ass^{n}s up to normality

$$\hat{\beta}_j \sim N(\beta_j, var(\hat{\beta}_j))$$

$$\frac{\hat{\beta}_j - \beta_j}{sd(\hat{\beta}_j)} \sim N(0, 1)$$

$$\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim t_{n-k-1} \quad (t \text{ dist. with } df = n-k-1)$$

$$\rightarrow N(0,1) \text{ as } df \rightarrow \infty$$



Single Hypothesis – Single Parameter

Null : $H_0 : \beta_j = a_j$ $\qquad a_j = 0 \longrightarrow$ special case

Alternative : $H_1 : \beta_j \neq a_j \longrightarrow$ two-tailed test

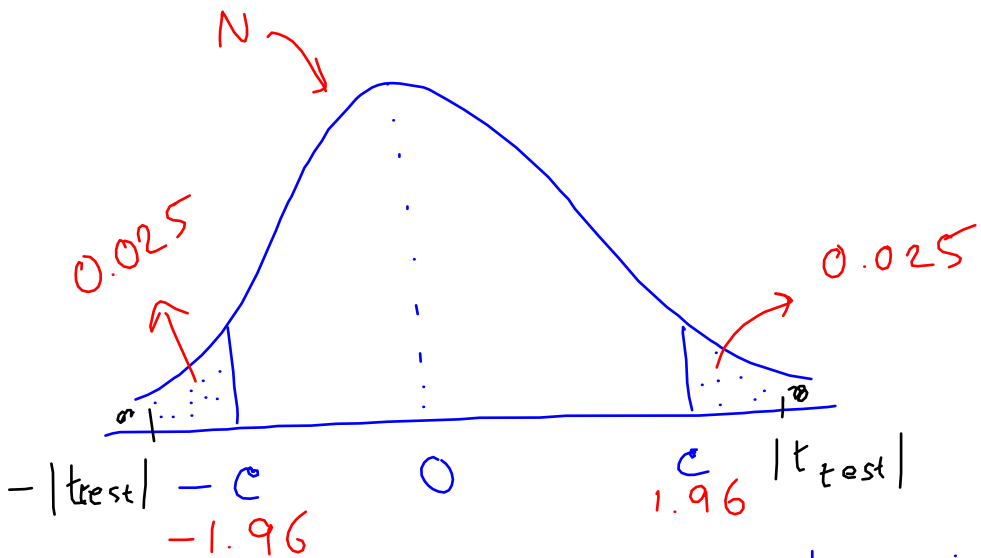
$>$ would be
$<$ one-tailed test

Test statistic
t "
t ratio

$$t_{test} = \frac{\hat{\beta}_j - a_j}{se(\hat{\beta}_j)}$$

If H_0 true unlikely $|t_{test}| > c$

$c \rightarrow$ critical value



area $\boxed{} = \alpha$

$\boxed{} = 0.05$

Rejection rule : reject H_0 if $|t_{test}| > c$
else fail to reject.

α = significance level
$= P(\text{rej. } H_0 \mid H_0 \text{ true})$

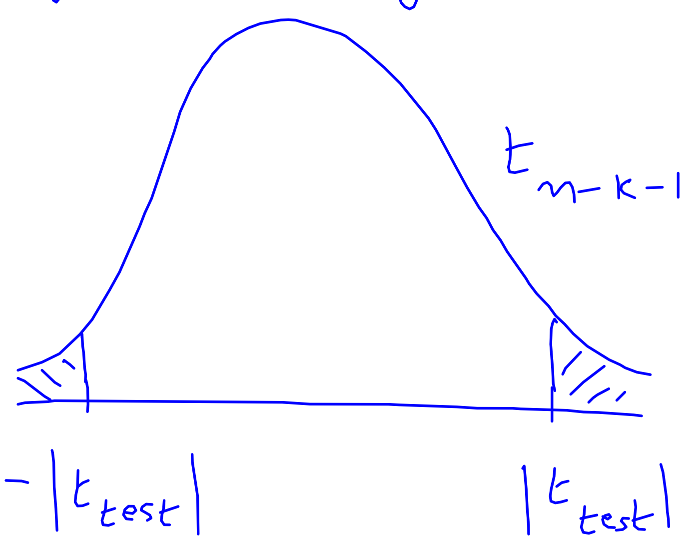
Equiv. rejection rule : reject H_0 if a_j is beyond $c \cdot se(\hat{\beta}_j)$ from $\hat{\beta}_j$

Fail to reject H_0 if a_j is within
$[\hat{\beta}_j - c \cdot se(\hat{\beta}_j), \ \hat{\beta}_j + c \cdot se(\hat{\beta}_j)]$

$(1-\alpha)$ conf. int. for β_j : $\hat{\beta}_j \pm c \cdot se(\hat{\beta}_j)$

$\downarrow$
conf. level ; $P(\text{not rej. } H_0 \mid H_0 \text{ true})$

Another equiv. rej. rule



$|t_{test}|$

Reject H_0 if area beyond $|t_{test}|$ &
$-|t_{test}| < \alpha$