

The Simple Regression Model: Definition, Estimation, and Statistical Properties

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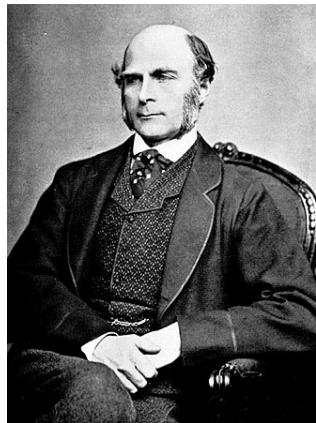
Randomized Trials

- Randomized trials

- ▶ https://www.youtube.com/watch?v=eGRd8jBdNYg&list=PL-uRhZ_p-BM5ovNRg-G6hDib270CvcyW8&index=4

Definition of the Simple Regression Model

Sir Francis Galton



https://en.wikipedia.org/wiki/Francis_Galton

Regression Towards Mediocrity in Hereditary Stature (1886)

Deriving the Ordinary Least Squares Estimates

Example

y (wage)	x (educ)
3.1	11
3.2	12
3	11
6	8
5.3	12
8.8	16
11	18
5	12
3.6	12
18	17

Deriving the Ordinary Least Squares Estimates (cont.)

$$\begin{aligned}\bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x} &= 0 \\ \overline{xy} - \hat{\beta}_0 \bar{x} - \hat{\beta}_1 \overline{x^2} &= 0\end{aligned}$$

$$\begin{aligned}6.742 - \hat{\beta}_0 - 12.9\hat{\beta}_1 &= 0 \\ 97.234 - 12.9\hat{\beta}_0 - 175.1\hat{\beta}_1 &= 0\end{aligned}$$

$$\begin{aligned}\hat{\beta}_0 &= -8.492 \\ \hat{\beta}_1 &= 1.181\end{aligned}$$

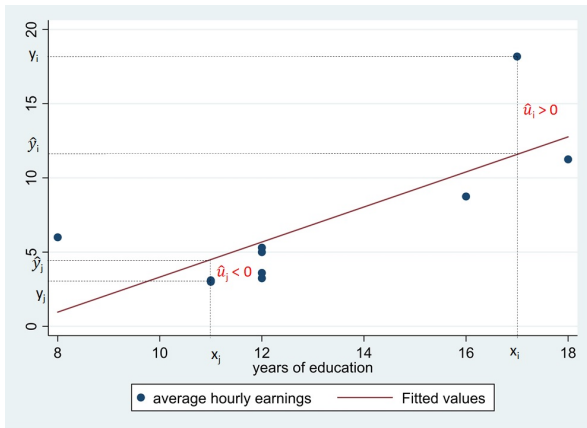
$\bar{x}$, $\bar{y}$, $\overline{xy}$, and $\overline{x^2}$: sample average of x , y , xy , and x^2

Deriving the Ordinary Least Squares Estimates (cont.)



<https://www.stata.com/giftshop/beta-hat/>

Deriving the Ordinary Least Squares Estimates (cont.)



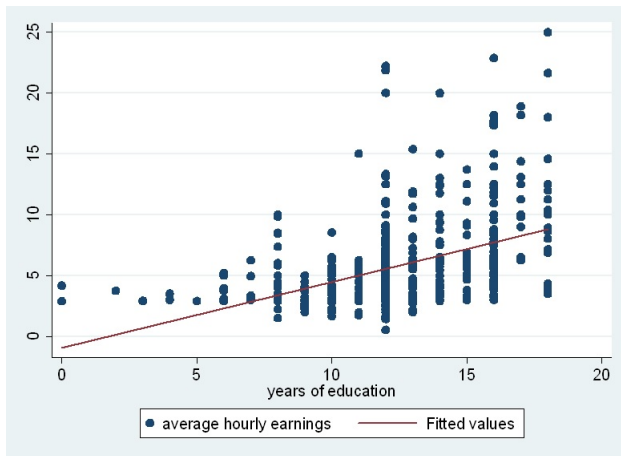
Deriving the Ordinary Least Squares Estimates (cont.)

y (wage)	x (educ)	$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$	$\hat{u} = y - \hat{y}$
3.1	11	4.498	-1.398
3.2	12	5.679	-2.439
3	11	4.498	-1.498
6	8	0.955	5.045
5.3	12	5.679	-0.379
8.8	16	10.403	-1.653
11	18	12.765	-1.515
5	12	5.679	-0.679
3.6	12	5.679	-2.079
18	17	11.584	6.596

Deriving the Ordinary Least Squares Estimates (cont.)

Data: wage1

- *wage*: dollars per hour; *educ*: years of education; $n = 526$
- Estimated equation: $\widehat{wage} = -0.90 + 0.54 \text{ educ}$

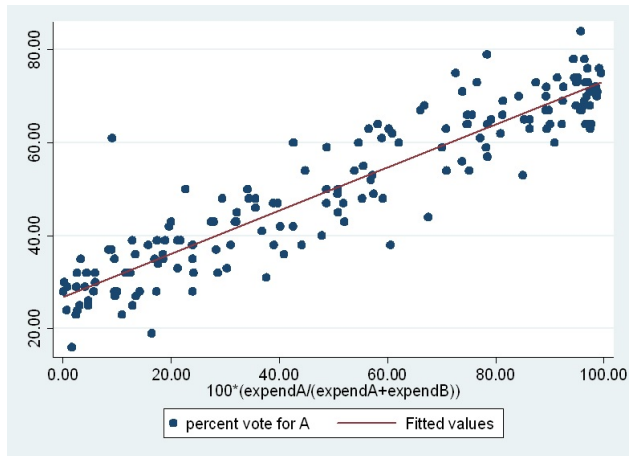


Properties of OLS

y (wage)	x (educ)	$\hat{y}$	$\hat{u}$	$x\hat{u}$	$\hat{y}\hat{u}$
3.1	11	4.498	-1.398	-15.381	-6.290
3.2	12	5.679	-2.439	-29.270	-13.852
3	11	4.498	-1.498	-16.481	-6.740
6	8	0.955	5.045	40.356	4.820
5.3	12	5.679	-0.379	-4.550	-2.153
8.8	16	10.403	-1.653	-26.446	-17.194
11	18	12.765	-1.515	-27.265	-19.335
5	12	5.679	-0.679	-8.150	-3.857
3.6	12	5.679	-2.079	-24.950	-11.808
18	17	11.584	6.596	112.136	76.409

Properties of OLS (cont.)

Data: vote1



Properties of OLS (cont.)

y (wage)	x (educ)	$\hat{y}$	$\hat{u}$	$(y - \bar{y})^2$	$(\hat{y} - \bar{y})^2$	$\hat{u}^2$
3.1	11	4.498	-1.398	13.264	5.034	1.955
3.2	12	5.679	-2.439	12.264	1.130	5.950
3	11	4.498	-1.498	14.003	5.034	2.245
6	8	0.955	5.045	0.551	33.484	25.447
5.3	12	5.679	-0.379	2.079	1.130	0.144
8.8	16	10.403	-1.653	4.032	13.402	2.732
11	18	12.765	-1.515	20.322	36.273	2.294
5	12	5.679	-0.679	3.035	1.130	0.461
3.6	12	5.679	-2.079	9.872	1.130	4.323
18	17	11.584	6.596	130.828	23.443	43.510

$$SST = 210.249 \quad SSE = 121.188$$

$$SSR = 89.061 \quad R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST} = 0.576$$