More on Functional Form

1. Quadratic functions
2. Interaction terms

Quadratic functions

Frankel and Rose (2005)

Industrial CO_2 Emissions (kt) vs. Real Per Capita GNP '87 int'l $
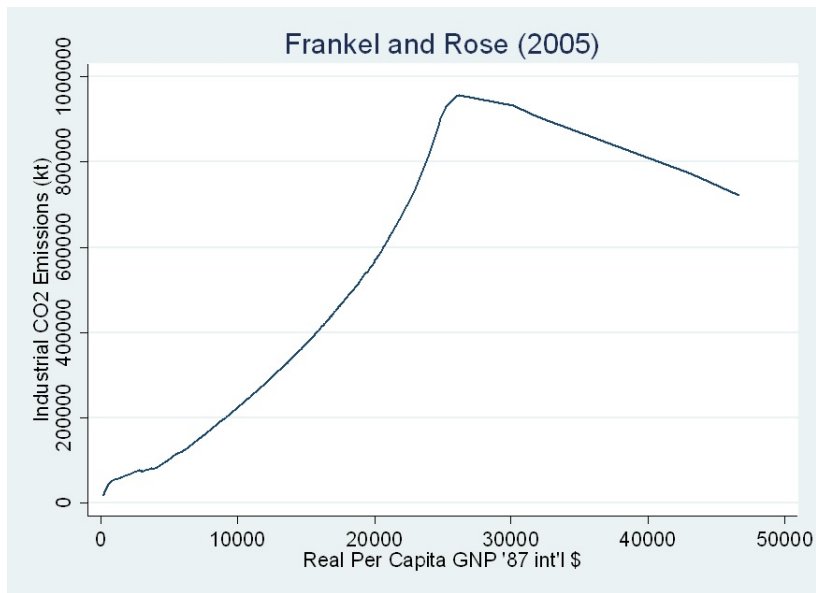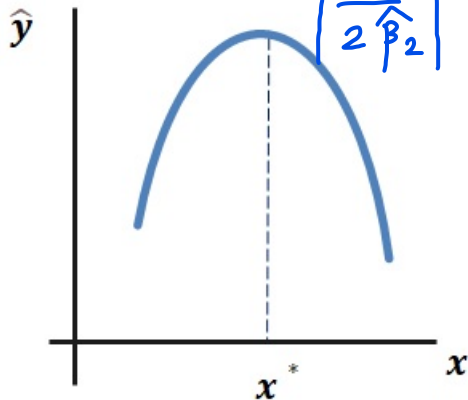
Quadratic functions (cont.)

- Increasing or decreasing effects

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$$
$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x$$

Quadratic functions (cont.)
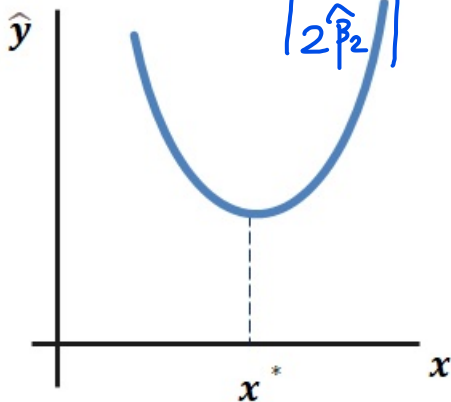
$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x$$

- $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 < 0$
  - ▸ Turning point at $x^* = \left| \dfrac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$

Quadratic functions (cont.)

- $\hat{\beta}_1 < 0$ and $\hat{\beta}_2 > 0$
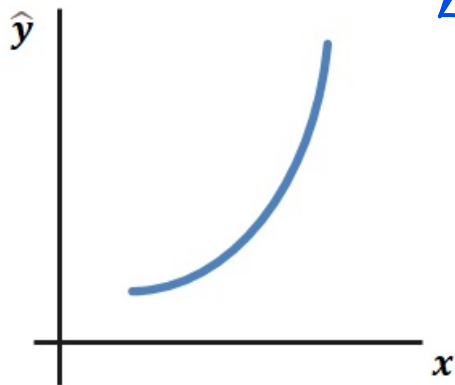 - ▸ Turning point at $x^* = \left| \dfrac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$

$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x$$

Quadratic functions (cont.)
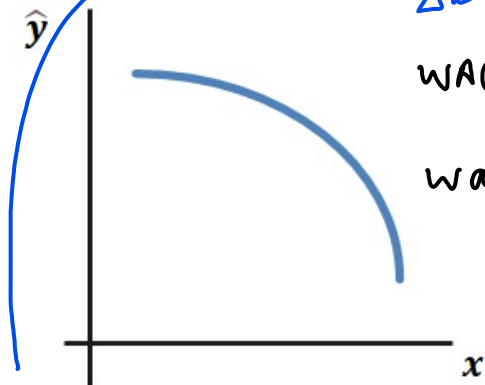
- $\hat{\beta}_1 > 0$ and $\hat{\beta}_2 > 0$

$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x$$

$= 0.176$ (at exp$\cdot$ $= .10$)
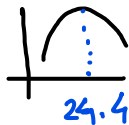
$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x$$

exp* $= 24.4$
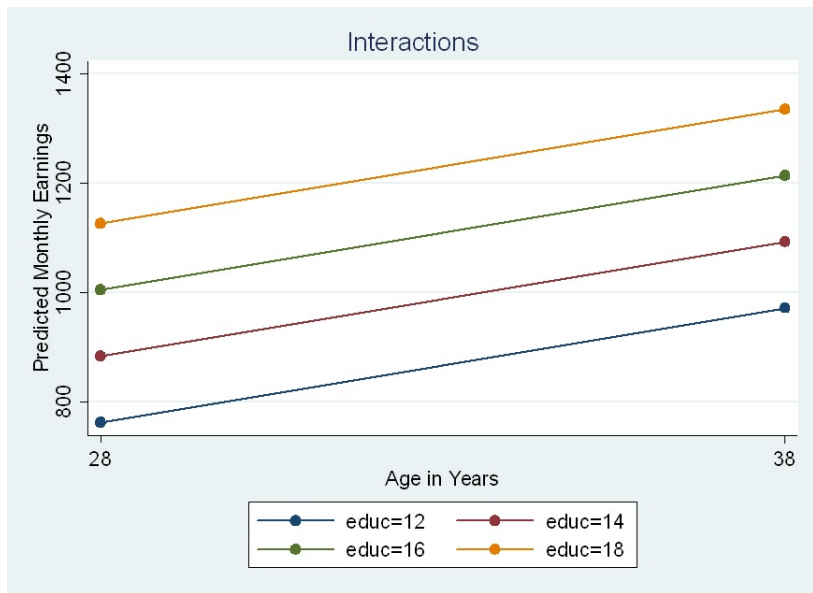
- $\hat{\beta}_1 < 0$ and $\hat{\beta}_2 < 0$

$\hat{y}$

x

WAGE 1:

24.4

$$wage = \beta_0 + \beta_1 \, exper + \beta_2 \, exper^2 + u$$

$$\frac{\Delta wage}{\Delta exper} = \beta_1 + 2\beta_2 \, exper$$

$= 0.298$ (at exp. $= 0$)

$\hat{\beta}_1 = 0.298$
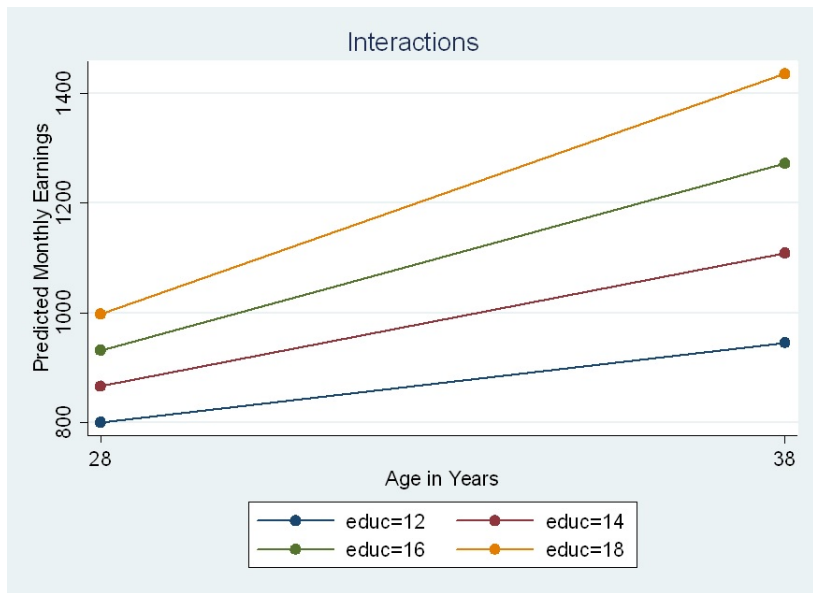
$\hat{\beta}_2 = -0.0061$

Interaction Terms

Interaction Terms (cont.)

Interaction Terms (cont.)

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 IQ + \beta_3 educ \times IQ + u$$

- Effect of one explanatory variable may depend on another
- Include interaction terms

$$\frac{\Delta \log(wage)}{\Delta educ} = \beta_1 + \beta_3 IQ$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + u$$

$$\frac{\Delta y}{\Delta x_1} = \beta_1 + \beta_3 x_2$$

$$\frac{\Delta y}{\Delta x_2} = \beta_2 + \beta_3 x_1$$

- Example estimates

$$= 101.28$$
avg.

$$\frac{\Delta y}{\Delta x_1} \text{ at } x_2 = \bar{x}_2 : \hat{\beta}_1 + \hat{\beta}_3 \bar{x}_2$$

$$\frac{\Delta y}{\Delta x_2} \text{ at } x_1 = \bar{x}_1 :$$

$$\hat{\beta}_2 + \hat{\beta}_3 \bar{x}_1$$

$$\hat{\beta}_1 = 0.025$$

$$\hat{\beta}_2 = 0.004$$

$$\hat{\beta}_3 = 0.0001$$

$$= 0.035$$

at $IQ = 101.28$