Asymptotics

1. Consistency
2. Asymptotic normality

Consistency

asymptotic bias under slightly weaker assumptions

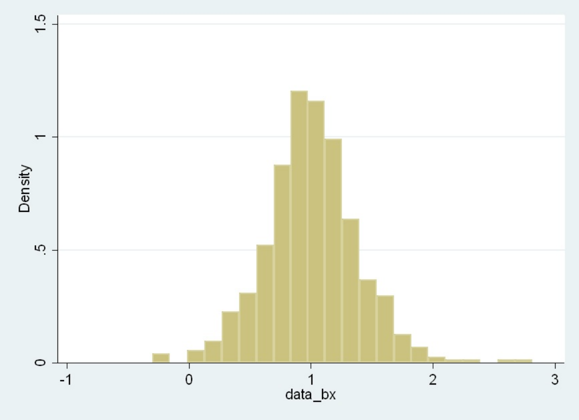
$E(u) = 0$

$corr(x, u) = 0$

- No
- For unbiased $\hat{\beta}_j$ → as $n \uparrow$ dist. of $\hat{\beta}_j$ more concentrated around β_j
- Example with normally distributed u

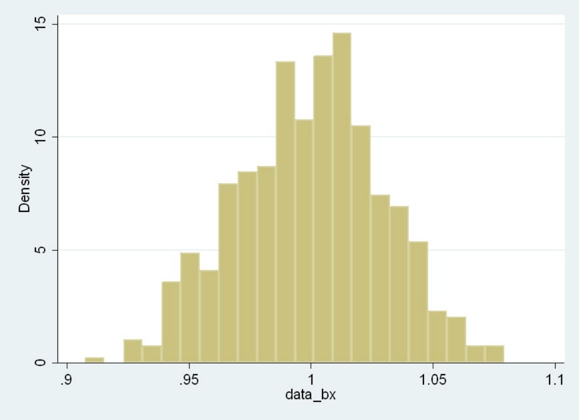
Consistency (cont.)

- $n = 10$, reps $= 500$, $x \sim N(0,1)$, $u \sim N(0,1)$
- $y = 1 + 1x + u$, distribution of $\hat{\beta}_1$



Consistency (cont.)

- $n = 1000$, reps $= 500$, $x \sim N(0,1)$, $u \sim N(0,1)$
- $y = 1 + 1x + u$, distribution of $\hat{\beta}_1$

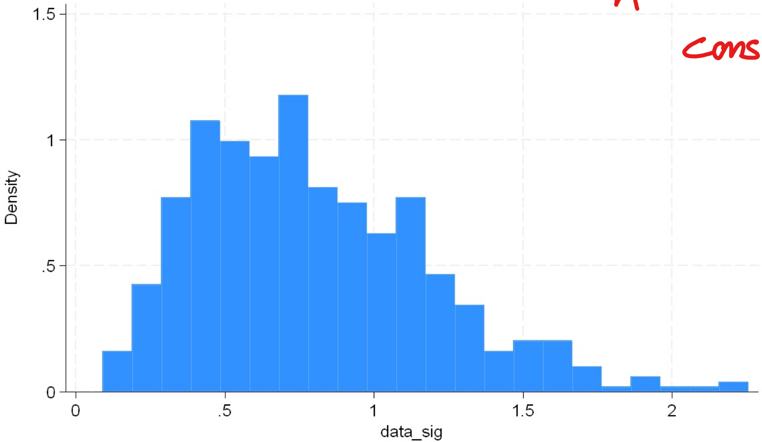


Consistency (cont.)

$\frac{SSR}{n-2}$ $\frac{SSR}{n-(k+1)}$: unbiased

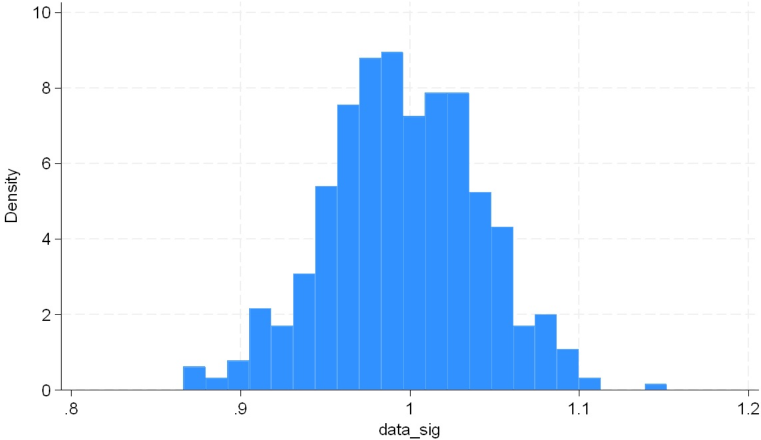
- $n = 10$, reps $= 500$, $x \sim N(0,1)$, $u \sim N(0,1)$
- $y = 1 + x + u$, distribution of SSR/n

$\frac{SSR}{n}$: biased but consistent



Consistency (cont.)

- $n = 1000$, reps $= 500$, $x \sim N(0, 1)$, $u \sim N(0, 1)$
- $y = 1 + x + u$, distribution of SSR/n

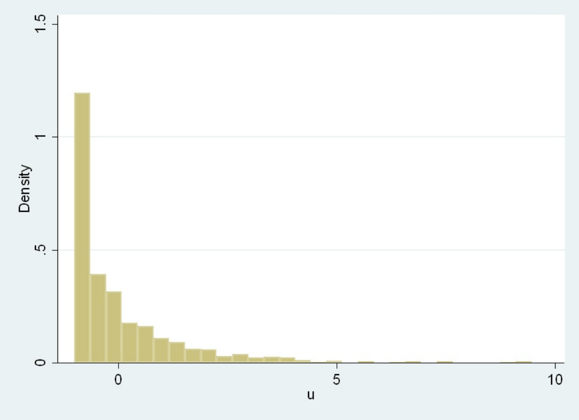


Asymptotic Normality

- MLR.6
  - Normal distribution of u
  - Normal distribution of y given $x_1, ..., x_k$
  - Often not the case

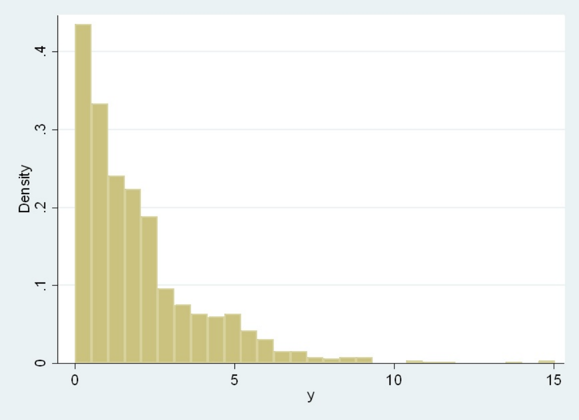
Asymptotic Normality (cont.)

- $u \sim \chi^2(1) - 1$



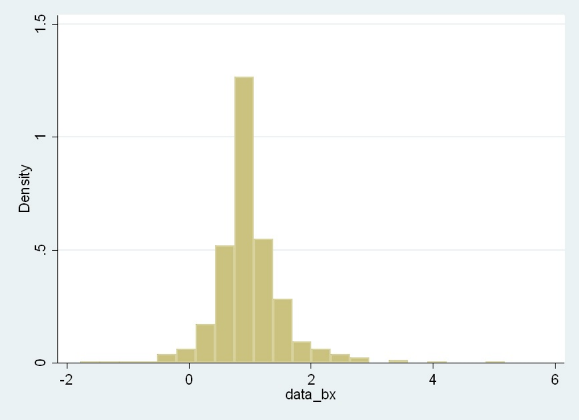
Asymptotic Normality (cont.)

- $x \sim \chi^2(1)$, $u \sim \chi^2(1) - 1$
- $y = 1 + x + u$



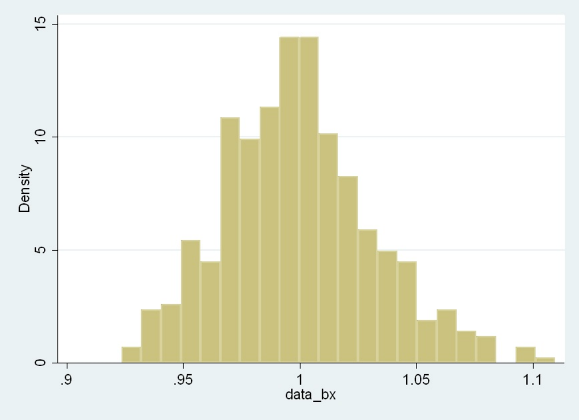
Asymptotic Normality (cont.)

- $n = 10$, reps $= 500$
- $x \sim \chi^2(1)$, $u \sim \chi^2(1) - 1$
- $y = 1 + x + u$



Asymptotic Normality (cont.)

- $n = 1000$, reps $= 500$
- $x \sim \chi^2(1)$, $u \sim \chi^2(1) - 1$
- $y = 1 + x + u$



Asymptotic Normality (cont.)

- Under MLR.1 to MLR.5

$$\frac{(\hat{\beta}_j - \beta_j)}{se(\hat{\beta}_j)} \overset{a}{\sim} Normal(0, 1)$$

- Can also write

$$\frac{(\hat{\beta}_j - \beta_j)}{se(\hat{\beta}_j)} \overset{a}{\sim} t_{n-k-1}$$

  - t_{df} approaches $Normal(0, 1)$ as df gets large
- F statistics also have approximate F distributions with large n