Instrumental Variables

1. IV Estimation in a Simple Regression Model
2. IV Estimation in a Multiple Regression Model

IV Estimation in a Simple Regression Model

instrumental variable or instrument for x

x and u are correl.

$\Rightarrow$ x is endog.

- Endogeneity
- One solution - https://youtu.be/eoJUPd6104Q $\Rightarrow$ OLS is biased
- Model

grade attendance

grade charter

wage educ.

$$y = \beta_0 + \beta_1 x + u$$

z: lottery

- Suppose z is a variable such that

corr. w/ x and

z: compulsory schooling laws

uncorr. w/ u and

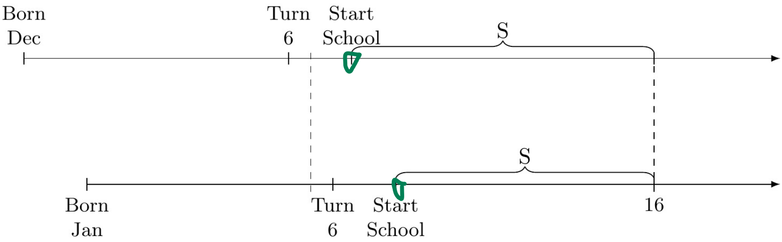
z: dist. from school

only affects y via x

then we may still be able to estimate β_1.

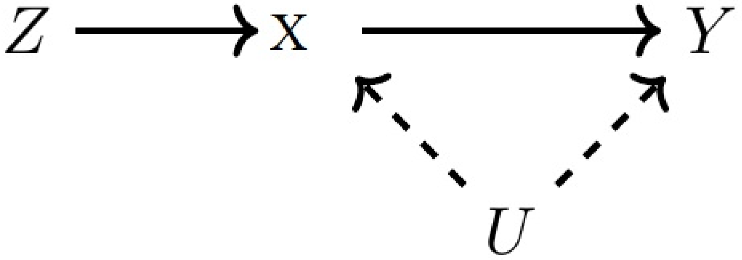
IV Estimation in a Simple Regression Model (cont.)

- Examples



Causal Inference: The Mixtape

IV Estimation in a Simple Regression Model (cont.)



Causal Inference: The Mixtape

IV Estimation in a Simple Regression Model (cont.)

- z and u are uncorrelated

- instrument exogeneity
- z has no direct effect on y (after controlling for x)
- z is uncorr. w/ omitted vars.
- often difficult to test

IV Estimation in a Simple Regression Model (cont.)

- z and x are correlated

– instrument relevance

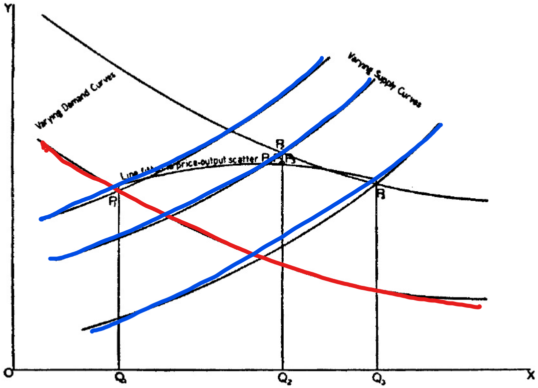
– z is related to x

– can be tested by estimating

$$x = \pi_0 + \pi_1 z + v$$

IV Estimation in a Simple Regression Model (cont.)

"Movements in demand and supply can produce an arbitrary scatterplot ... which will trace out neither supply nor demand unless one of the curves is fixed."



Stock and Trebbi (2003)

IV Estimation in a Simple Regression Model (cont.)

- Model

$$y = \beta_0 + \beta_1 x + u$$

  - x and u : correlated
  - z is an instrument for x

- The *instrumental variables* (*IV*) *estimator* of β_1

$z = x \to$ OLS formula

$$\widehat{\beta}_1 = \frac{\sum_i (z_i - \overline{z})(y_i - \overline{y})}{\sum_i (z_i - \overline{z})(x_i - \overline{x})}$$

$\boxed{\text{effect of } z \text{ on } x}$ × effect of x on y = $\boxed{\text{effect of } z \text{ on } y}$

IV Estimation in a Simple Regression Model (cont.)

- If z and u are uncorrelated, and z and x are correlated, the IV estimator is consistent
- The IV estimator is never unbiased (large samples preferred)
- In small samples, the IV estimator can have substantial bias

IV Estimation in a Simple Regression Model (cont.)

- Statistical Inference
  - Homoskedasticity assumption

$$E(u^2|z) = \sigma^2 = Var(u)$$

  - Asymptotic standard error of $\hat{\beta}_1$ $\sqrt{\frac{\hat{\sigma}^2}{SST_x R^2_{xz}}}$

where SST_x is the total sum of squares of x
$R^2_{x,z}$: R-squared from the regression of x on z → 1st stage
$\hat{\sigma}^2 = \frac{1}{n-2}\sum_i \widehat{u}_i^2$
$\widehat{u}_i$: IV residuals

IV Estimation in a Simple Regression Model (cont.)

- Note
  - Standard error of $\hat{\beta}_1$ in case of OLS $\sqrt{\dfrac{\hat{\sigma}^2}{SST_x}}$

    where SST_x is the total sum of squares of x
    $\hat{\sigma}^2 = \frac{1}{n-2}\sum_i \widehat{u}_i^2$
    $\widehat{u}_i$: OLS residuals
  - Typically $R^2_{x,z} < 1$ and the IV std. error is always larger than the OLS std. err.
  - If x and z are only slightly correlated

$R^2_{x,z}$ is small and the IV std. error is large.

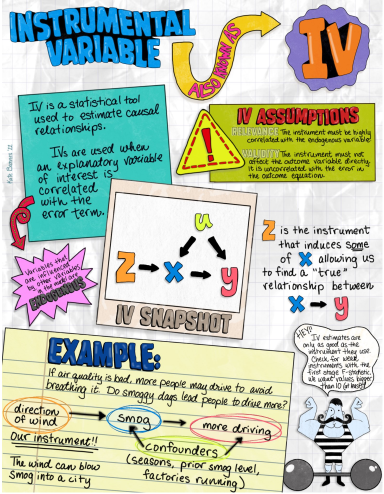
IV Estimation in a Simple Regression Model (cont.)

weak instruments

- Note (cont.)
  - If the correlation between x and z is low, we have the problem of
    - ★ Inconsistency in the IV estimator related to $\frac{Corr(z,u)}{Corr(z,x)}$
    - ★ This inconsistency (asymptotic bias) in the IV estimator can be large

even if z and u are slightly correlated.

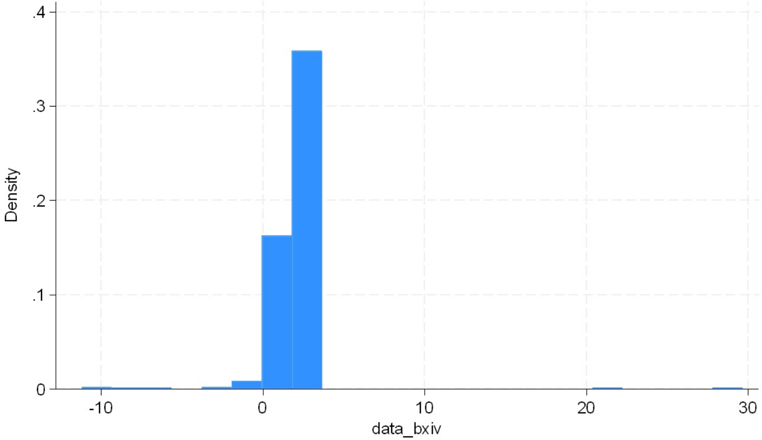
IV Estimation in a Simple Regression Model (cont.)



@Kate__Barnes

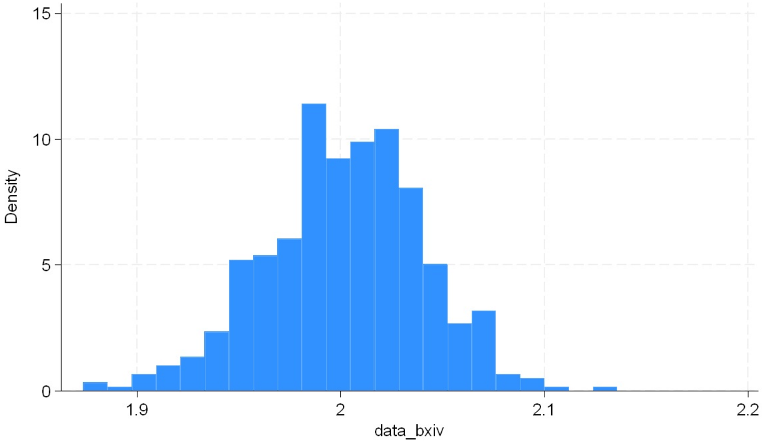
IV Estimation in a Simple Regression Model (cont.)

- $n = 10$, reps $= 500$, corr$(x, z = 0.8)$, corr$(x, u = 0.5)$, corr$(z, u = 0)$
- $y = 1 + 2x + u$



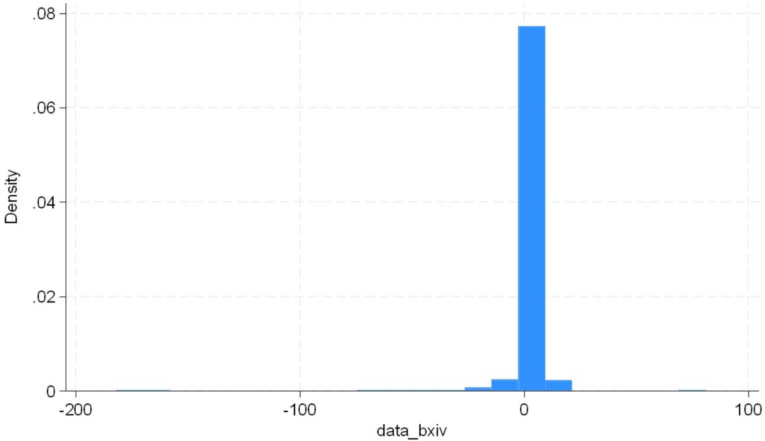
IV Estimation in a Simple Regression Model (cont.)

- $n = 1000$, reps $= 500$, corr$(x, z = 0.8)$, corr$(x, u = 0.5)$, corr$(z, u = 0)$
- $y = 1 + 2x + u$



IV Estimation in a Simple Regression Model (cont.)

- $n = 1000$, reps $= 500$, corr$(x, z = 0.01)$, corr$(x, u = 0.5)$, corr$(z, u = 0)$
- $y = 1 + 2x + u$



IV Estimation in a Multiple Regression Model

- Model

$$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 z_1 + u_1$$

  - Sometimes called
  - y_1 correlated with u_1 $\Rightarrow$
  - z_1 assumed to be uncorrelated with u_1 $\Rightarrow$
  - y_2 correlated with u_1 $\Rightarrow$
  - OLS estimators:
  - z_2 instrumental variable for y_2

IV Estimation in a Multiple Regression Model (cont.)

- Assumptions

$$
\begin{aligned}
E(u_1) &= \\
E(z_1 u_1) &= \\
E(z_2 u_1) &=
\end{aligned}
$$

IV Estimation in a Multiple Regression Model (cont.)

- Equations

$$\begin{aligned} E(y_1 - \beta_0 - \beta_1 y_2 - \beta_2 z_1) &= \\ E\left[z_1(y_1 - \beta_0 - \beta_1 y_2 - \beta_2 z_1)\right] &= \\ E\left[z_2(y_1 - \beta_0 - \beta_1 y_2 - \beta_2 z_1)\right] &= \end{aligned}$$

IV Estimation in a Multiple Regression Model (cont.)

- Sample analogs

$$
\begin{aligned}
n^{-1} \sum_{i=1}^{n} \left(y_{i1} - \widehat{\beta}_0 - \widehat{\beta}_1 y_{i2} - \widehat{\beta}_2 z_{i1} \right) &= \\
n^{-1} \sum_{i=1}^{n} z_{i1} \left(y_{i1} - \widehat{\beta}_0 - \widehat{\beta}_1 y_{i2} - \widehat{\beta}_2 z_{i1} \right) &= \\
n^{-1} \sum_{i=1}^{n} z_{i2} \left(y_{i1} - \widehat{\beta}_0 - \widehat{\beta}_1 y_{i2} - \widehat{\beta}_2 z_{i1} \right) &=
\end{aligned}
$$

- *Instrumental variables estimators*

IV Estimation in a Multiple Regression Model (cont.)

- Still need z_2 and y_2 to be
- *Reduced form equation*

$$y_2 = \pi_0 + \pi_1 z_1 + \pi_2 z_2 + v_2$$

  - Key condition

IV Estimation in a Multiple Regression Model (cont.)

- Note
  - Structural equation

$$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 z_1 + u_1$$

  - Substituting

$$y_2 = \pi_0 + \pi_1 z_1 + \pi_2 z_2 + v_2$$

  - Reduced form

$$\begin{aligned} y_1 &= \beta_0 + \beta_1 (\pi_0 + \pi_1 z_1 + \pi_2 z_2 + v_2) + \beta_2 z_1 + u_1 \\ y_1 &= (\beta_0 + \beta_1 \pi_0) + (\beta_1 \pi_1 + \beta_2) z_1 + \beta_1 \pi_2 z_2 + (u_1 + \beta_1 v_2) \end{aligned}$$

  - Need IV to estimate