

Ch. 3 (cont.)

More complicated derivation of bias
w/ addl. explanatory vars.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

$\downarrow$ FDI $\downarrow$ env. reg. $\downarrow$ input prices $\downarrow$ infrastruc. $\rightarrow$ political activism

If x_1 corr. w/ u

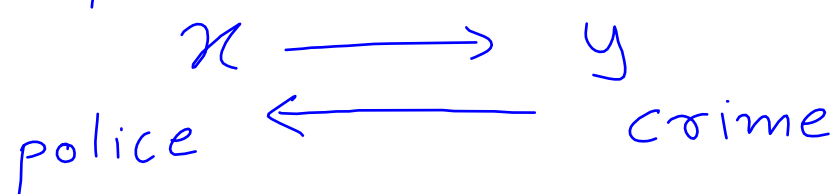
κ_2, κ_3 not corr. w/ u

OLS estimator biased for all β 's if x_1 corr. w/ x_2 & x_3

Other sources of bias

Measurement error in x or y
(e.g., crime, distance, ...)

Simultaneity



Sample selection

data observed if $y > \text{threshold}$
(e.g. trade)

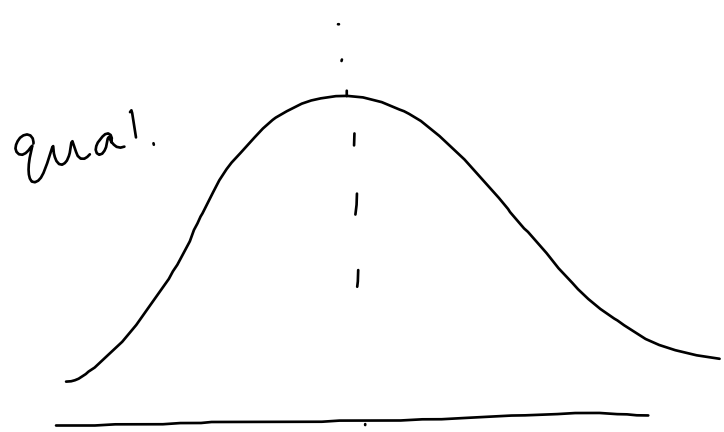
Inclusion of irrelevant regressors

- exercise caution e.g.

y	x
#accidents	alcohol laws, <u>alcohol consumption?</u>
wage	discrim., educ, <u>occupation?</u>

Variance

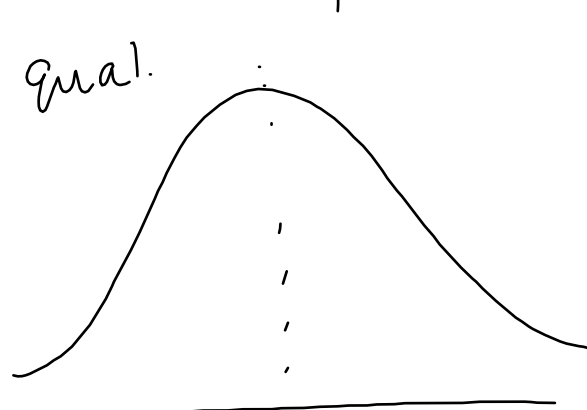
$$\begin{aligned} \text{property value} &= \beta_0 + \beta_1 \text{sq.ft.} + \beta_2 \# \text{BRs} \\ &\quad + \beta_3 \text{lot size} + u \end{aligned}$$



size = 1500 sq.ft.

#BRs = 2

lot size = 2000 sq.ft.



size = 2500 sq.ft.

#BRs = 3

lot size = 3000 sq.ft.

$$\text{var}(\text{quality}) = \text{Var}(\text{quality})$$

$$k = 3 \quad y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u_{\text{quality}}$$

price sq.ft. #BRs lot size

Homoskedasticity

$$\text{var}(u | x_1, x_2, x_3) = \sigma^2$$

$$\text{var}(\hat{\beta}_j) = \frac{\sigma^2}{SST_j (1 - R_j^2)} \quad j = 1, 2, 3$$

$$SST_j = \sum_{i=1}^n (x_{ij} - \bar{x}_j)^2$$

R_j^2 : R^2 from reg. of x_j on other x 's

y : price x_1 : sq.ft. x_2 : #BRs x_3 : lot size

$$\text{var}(\hat{\beta}_1) = \frac{\sigma^2}{\text{Total variation in sq.ft.} (1 - R^2 \text{ from reg. of sq.ft. on } \# \text{BRs} \times \text{lot size})}$$

dep. var. indep. vars.

R_j^2 close to 1 $\rightarrow$ multicollinearity
 $\rightarrow$ does not violate assⁿs for unbiasedness

Under the assⁿs up to homosk.

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^n \hat{u}_i^2}{n - k - 1}$$

↓
unbiased estimator of σ^2

↓
of regressors

e.g. $k = 3$
 $\Rightarrow n - k - 1 = n - 4$

$$se(\hat{\beta}_j) = \frac{\hat{\sigma}}{\sqrt{SST_j (1 - R_j^2)}} \quad j = 1, 2, 3$$