Ch 7 Dummy / Binary Variables

Binary / dummy indep vars

y	x	u
trade / env performance	trade / env agreement	political preferences
wage	marital status	work ethic, reliability

Single dummy variable

$$y = \beta_0 + \delta_0 x + u$$

$$\hookrightarrow 0/1 \text{ indicator}$$

$$E(y|x) = \beta_0 + \delta_0 x$$

$$\Rightarrow \quad \delta_0 = E(y|x=1) - E(y|x=0)$$

effect of $x=1$ rel. to $x=0$ (base/reference group)

$$\hat{\delta}_0 = \bar{y}_{x=1} - \bar{y}_{x=0}$$

$$= \overline{wage}_{marr} - \overline{wage}_{not\ marr}$$

if $y \to$ wage

$x \to 1$ for marr

0 for not marr

$$y = \beta_0 + \beta_1 x_1 + \delta_0 x_2 + u$$

$\quad\quad\quad\quad\quad\quad \hookrightarrow 0/1 \quad$ binary indicator

if $y \to$ wage

$x_1 \to$ educ

$$E(y | x_1, x_2) = \beta_0 + \beta_1 x_1 + \delta_0 x_2$$

$x_2 \to 1$ marr

0 not marr

$$\delta_0 = E(y | x_1, x_2 = 1) - E(y | x_1, x_2 = 0)$$

base/reference group

effect of $x_2 : 1$ rel to $x_2 = 0$ after controlling for x_1

Note   2 groups denoted by single   dummy $x_2 + x_3 = 1$

$$x_3 \longrightarrow 1 \quad \text{not marr}$$
$$0 \quad \text{marr}$$

x_2	x_3
1	0
0	1

Not reqd x_2 and x_3 are perfectly   collinear

(dummy var. trap)

eg $\hat{\delta_0} = 0.26$

$\Rightarrow 26 \,/$

If   y   log(wage)

approximate effect of $x_2 = 1$ $100 \,\hat{\delta_0} \,/.$

Exact   " $100 \left[\exp(\hat{\delta_0}) - 1 \right] /$ $\Rightarrow 29.7 \,/$

Multiple categories

$$M \rightarrow \begin{cases} 1 \ (\text{married}) \\ 0 \ (\text{not } ") \end{cases}$$

$$W \rightarrow \begin{cases} 1 \ (\text{western region}) \\ 0 \ (\text{not } ") \end{cases}$$

4 groups $\rightarrow$ choose 1 as base/reference &
include dummies for the rest

$$y = \beta_0 + \beta_1 x_1 + \delta_0 x_2 + \delta_1 x_3 + \delta_2 x_4 + u$$

if   y   wage

      x_1 educ

x_2 $M = 1$ & $W = 1 \rightarrow x_2 = 1$ & 0 ow

x_3 $M = 1$ & $W = 0 \rightarrow x_3 = 1$ & 0 ow

x_4 $M = 0$ & $W = 1 \rightarrow x_4 = 1$ & 0 ow.

      base $M = 0$ & $W = 0$

δ_0 effect of $x_2 = 1$ rel to base

δ_1 "   " $x_3 = 1$ "

δ_2 "   " $x_4 = 1$

e g $\hat{\delta}_0 = 272$ $\hat{\delta}_1 = 13$ $\hat{\delta}_2 = 0.02$

Effect of marr & western rel to base $= \$ 272$

" " " & not " " " $= \$ 13$

Interactions among dummy vars

$$y = \beta_0 + \beta_1 x_1 + \delta_M M + \delta_w W + \delta_{Mw} M \times W + u$$

Base / reference group $M = 0, W = 0$

Effect of $M = 1$ & $W = 1$ $\delta_M + \delta_w + \delta_{Mw}$ Effect of $M = 0$ & $W = 1$ rel to base
rel to base δ_w

" $M = 1$ & $W = 0$ " δ_M

$\hat{\delta}_M = 1.3$

$\hat{\delta}_W = 0.02$

$\hat{\delta}_{MW} = 1.4$

Effect of marr. & west rel to base $= \$2.72$

" marr & not west " $= \$1.3$

Allowing for diff slopes

$$y = \beta_0 + \beta_1 x_1 + \delta_0 M + \delta_1 M \cdot x_1 + u$$

if y wage
x_1 educ

$M \rightarrow$ 1 for marr
 0 " not "

	Intercept	Slope for educ
Marr	$\beta_0 + \delta_0$	$\beta_1 + \delta_1$
Not marr	β_0	β_1
Diff	δ_0	δ_1

e g $\hat{\beta_1} = 0 \ 46$

$\hat{\delta_0} = 0 \ 33$

$\hat{\delta_1} = 0 \ 1$