Additional Panel Data Methods

1. Fixed Effects Estimation
2. Random Effects Models

Fixed Effects Estimation

- Alternative to first differencing
- Model

$$y_{it} = \beta_0 + \beta_1 x_{it} + a_i + u_{it}$$

- Averaging over each i

$$\bar{y}_i = \beta_0 + \beta_1 \bar{x}_i + a_i + \bar{u}_i$$

- Subtracting

$$y_{it} - \bar{y}_i = \beta_1 (x_{it} - \bar{x}_i) + u_{it} - \bar{u}_i$$
$$\ddot{y}_{it} = \beta_1 \ddot{x}_{it} + \ddot{u}_{it}$$

Fixed Effects Estimation (cont.)

- Example

$$crime_{it} = \beta_0 + \beta_1 unem_{it} + city_i + u_{it}$$

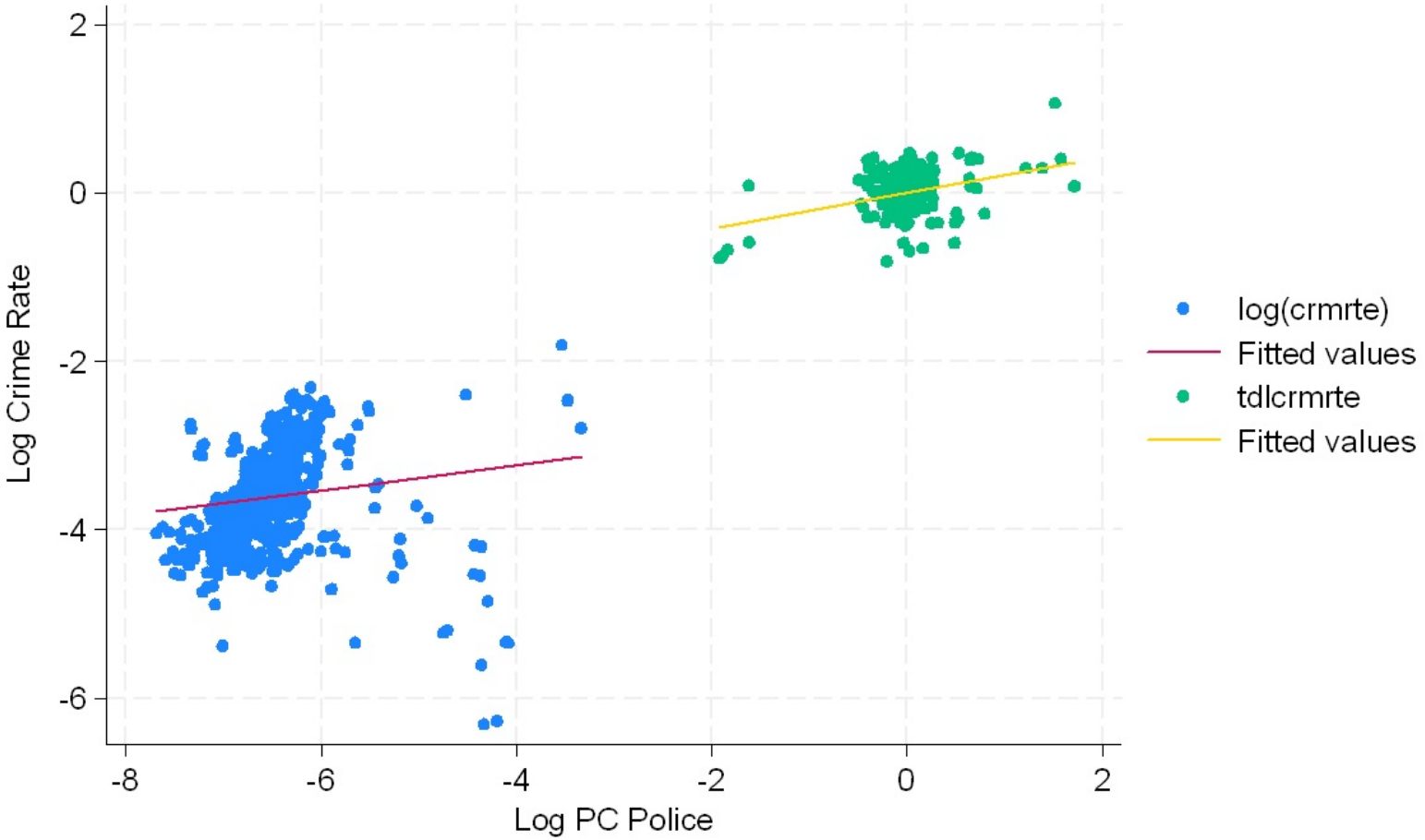
- Averaging over time

$$\overline{crime}_i = \beta_0 + \beta_1 \overline{unem}_i + city_i + \overline{u}_i$$

- Subtracting

$$crime_{it} - \overline{crime}_i = \beta_1 (unem_{it} - \overline{unem}_i) + u_{it} - \overline{u}_i$$
$$\ddot{crime}_{it} = \beta_1 \ddot{unem}_{it} + \ddot{u}_{it}$$

Fixed Effects Estimation (cont.)



Fixed Effects Estimation (cont.)

- Time-demeaned variables: $\ddot{y}_{it}, \ddot{x}_{it}$, and $\ddot{u}_{it}$
  - Also called the fixed effects/ within transformation
- OLS estimator based on the time-demeaned variables: fixed effects estimator

Relies on time variation in y and x within each cross-sectional unit

Fixed Effects Estimation (cont.)

arbitrary correl^n b/w any x_{ijt} & a_i

- Note
  - Fixed effects estimator allows for
  - Fixed effects transformation eliminates → any explanatory var. that is constant over time
  - Strict exogeneity $\Rightarrow$ u_{it} should be uncorrelated with each x_{ijt} across all time periods
    - ★ Example

$$crime_{it} = \beta_0 + \beta_1 unem_{it} + city_i + u_{it}$$

Under strict exog. → fixed effects estimator is unbiased

such vars. are controlled for nonetheless

Fixed Effects Estimation (cont.)

- Note (cont.)
  - Dummy variable regression

$$y_{it} = \beta_0 + \beta_1 x_{it} + a_i + u_{it}$$

  - ★ Include a dummy variable for each cross-sectional unit
  - ★ Results in the same estimates of β_1 (instead of FE/within)

Fixed Effects Estimation (cont.)

- Note (cont.)
  - Fixed effects versus first-differenced estimators
    - ★ For $T = 2$, both are identical
    - ★ For $T \geq 3$, the two are not identical, but both
    - ★ Inference (standard errors)

depends on ass^{n}s related to u_{it} and Δu_{it}

unbiased/consis-tent under ass^{n}s such as strict exog.

Fixed Effects Estimation (cont.)

all yrs. or pds. of data are observed for all cross-sectional units

- Note (cont.)
  - Balanced versus unbalanced panel
    - ★ Unbalanced: certain yrs. or pds. of data are missing for some cross-sectional units
    - ★ Balanced:
  - Unbalanced panel:

↓

unbiased & consistent provided reason for missing data not correl. with u_i it

Random Effects Models

- Model

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \ldots + \beta_k x_{itk} + a_i + u_{it}$$

- a_i assumed to be uncorr. w/ each x_{ijt} in all time pds.
- Composite error: $v_{it} = a_i + u_{it}$ - corr. across time
- Estimation via feasible generalized least squares (FGLS)
- Note: allows time-invariant explanatory vars.