

Ch. 5 :

Consistency

(in large samples)
No asymptotic bias
under slightly weaker
assⁿs

$$E(u) = 0$$

$$\text{corr}(x, u) = 0$$

For unbiased $\hat{\beta} \rightarrow$ as $n \uparrow$

distⁿ of $\hat{\beta}$ more concentrated
around true value

$\frac{SSR}{n - (k+1)} \rightarrow$ unbiased estimator of σ^2

$\frac{SSR}{n} \rightarrow$ biased but consistent

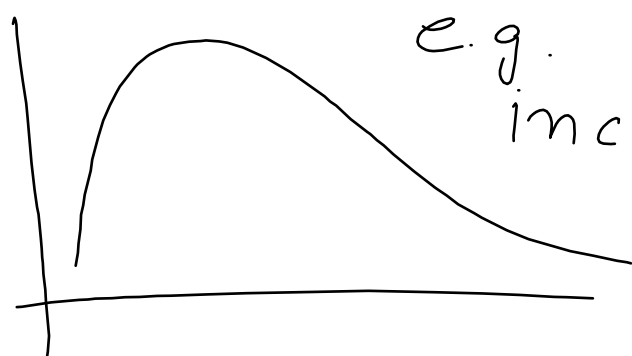
Asymptotic Normality

$$y = \beta_0 + \beta_1 x + u$$

$\uparrow$
 β

$\downarrow$
 u

Normal?



e.g.

income, property values

$u \rightarrow$ Normal

$y | x_1, \dots, x_k \rightarrow$ Normal

often not the case

e.g. $y \rightarrow$ binary indicator of
employment, graduation, ...

under the assⁿs for unbiasedness & homosk.

$$\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \stackrel{a}{\sim} \text{Normal}(0, 1)$$

$$\stackrel{a}{\sim} t_{n-k-1}$$

Ch.6: Quadratic Functions

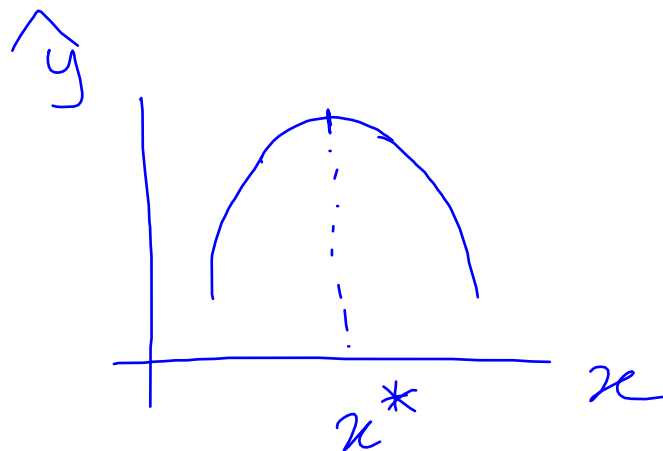
$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$$

$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 \cdot x$$

$$\hat{\beta}_1 > 0 \quad \& \quad \hat{\beta}_2 < 0$$

Turning pt. (max.)

$$\text{at } x^* = \left| \frac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$$



BWGH12

$$\text{bwght} = \beta_0 + \beta_1 \text{mage} + \beta_2 \text{mage}^2 + u$$

$$\hat{\beta}_1 = 97.36$$

$$\hat{\beta}_2 = -1.57$$

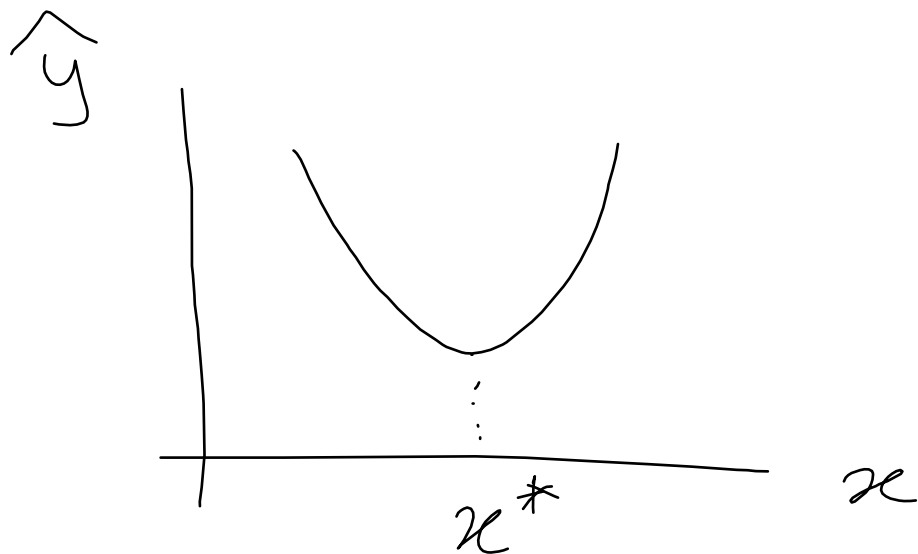
$$\frac{\Delta \text{bwght}}{\Delta \text{mage}} = \beta_1 + 2\beta_2 \text{mage}$$

$$\text{at } \text{mage} = 25 : \quad 97.36 + 2(-1.57)25 = 18.86$$

$$\text{at } \text{mage} = 40 : \quad 97.36 + 2(-1.57)40 = -28.24$$

$$\text{mage}^* = \left| \frac{97.36}{2(-1.57)} \right| = 31.03$$

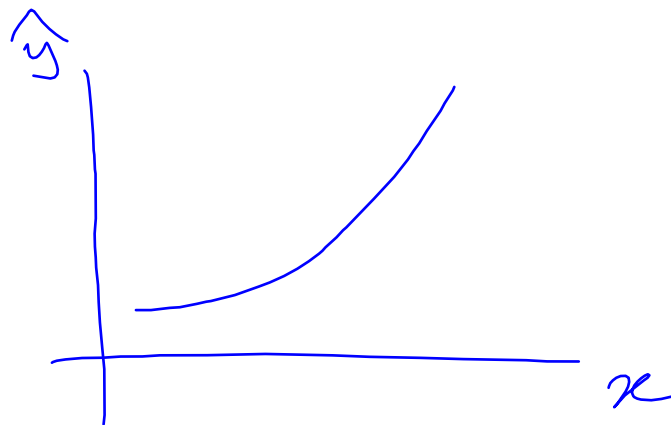
$$\hat{\beta}_1 < 0 \quad \& \quad \hat{\beta}_2 > 0$$



Turning pt. at $x^* = \left| \frac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$

$$\hat{\beta}_1 > 0$$

$$\hat{\beta}_2 > 0$$



$$\hat{\beta}_1 < 0$$

$$\hat{\beta}_2 < 0$$

