

Ch. 8 : Heteroskedasticity

Consequences :

$$\text{var}(u | x_1, \dots, x_k) \neq \sigma^2$$

but depends on x_j

$$\text{Wage} = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{major} + \beta_3 \text{exper} + u$$

$\text{var}(u | \text{educ}, \text{major}, \text{exper})$ depends on
major

$\text{var}(\text{wage} | \text{educ}, \text{major}, \text{exper})$ "

Usual std. errors & test statistics
no longer valid.

Heteroskedasticity - robust inference

Heterosk. of unknown form

"

- robust std. errors $\rightarrow$ available
in case of large samples

Simple regression:

$$se(\hat{\beta}_1) = \frac{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \hat{u}_i^2}}{SST_x}$$

$$\sum_{i=1}^n (x_i - \bar{x})^2$$

$\swarrow$

Multiple regression:

$$se(\hat{\beta}_j) = \frac{\sqrt{\sum_{i=1}^n \hat{r}_{ij}^2 \hat{u}_i^2}}{SST_j (1 - R_j^2)}$$

$\swarrow$
 $se(\hat{\beta}_1)$ i.e.
 $\hat{\beta}$ for
educ.

$$\sum_{i=1}^n (x_{ij} - \bar{x}_j)^2$$

total variation
in educ.

$\searrow$
 R^2 from reg. of
educ.
 x_j on all other
 x 's

major, exper.

$\hat{r}_{ij}$: residual from reg. of x_j on
all other x 's for obs i .

major, exper

Testing

Model

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + u$$

Null hyp. of homosk.

$$H_0 : \text{Var}(u | x_1, \dots, x_k) = \sigma^2$$

$$\text{or, } E(u^2 | x_1, \dots, x_k) = \sigma^2$$

Suppose

$$u^2 = \delta_0 + \delta_1 x_1 + \dots + \delta_k x_k + v$$

$$H_0 : \begin{array}{l} \delta_1 = 0 \\ \vdots \\ \delta_k = 0 \end{array}$$

can estimate

$$\hat{u}^2 = \delta_0 + \delta_1 x_1 + \dots + \delta_k x_k + v$$

Joint test in H_0 .

can also include squares, ^{x_1^2} interactions, ^{x_1, x_2}

& fitted ^{$\hat{y}$} values as indep. vars. in the reg. above.

Works only in case of large samples.

Weighted Least Squares

$$\text{Model : } y = \beta_0 + \beta_1 x_1 + \dots + \beta_k + u$$

$$\text{Var}(u | x_1 \dots x_k) = \sigma^2 h$$

Transformed model :

$$\frac{y}{\sqrt{h}} = \beta_0 \cdot \frac{1}{\sqrt{h}} + \beta_1 \cdot \frac{x_1}{\sqrt{h}} + \dots + \beta_k \frac{x_k}{\sqrt{h}} + \frac{u}{\sqrt{h}}$$

$$\text{Var}\left(\frac{u}{\sqrt{h}} \mid x_1 \dots x_k\right) = \frac{\sigma^2 h}{h} = \sigma^2$$

Generalized / weighted least squares.

Feasible / estimated GLS (FGLS / EGLS)
 $\therefore h$ must be estimated.