Ch. 17: Binary Dependent Variables

Latent variable framework:

Latent (unobserved) var.

$$y^* = \beta_0 + \beta_1 x + u$$

such that

$$y = 0 \quad \text{if} \quad y^* < 0$$
$$\;\; = 1 \quad \text{if} \quad y^* \geq 0$$

$$\beta_0 + \beta_1 x + u \geq 0$$

or, $u \geq -\beta_0 - \beta_1 x$

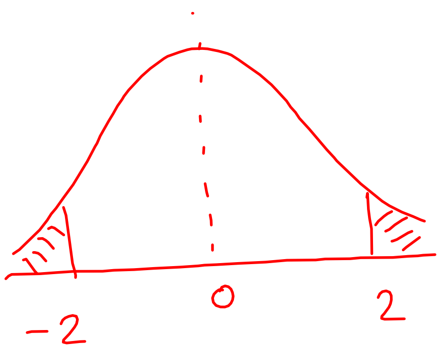
$$\Pr(y=1 \mid x) = \Pr(u \geq -\beta_0 - \beta_1 x)$$

↳ bounded b/w 0 & 1

Probit

$$u \sim N(0,1)$$

$$\begin{aligned}
\Pr(y=1 \mid x) &= \Pr(u \geq -\beta_0 - \beta_1 x) \\
&= \Pr(u \leq \beta_0 + \beta_1 x) \\
&= \Phi(\beta_0 + \beta_1 x)
\end{aligned}$$



$\hat{\beta}_0, \hat{\beta}_1$: from maximum likelihood estimation (MLE)

Logit

$u \sim$ logistic distribution

$$Pr(u \le \beta_0 + \beta_1 x)$$

$$= \frac{\exp(\beta_0 + \beta_1 x)}{1 + \exp(\beta_0 + \beta_1 x)}$$

$\hat{\beta}_0, \hat{\beta}_1$: from MLE.

Maximum Likelihood Estimation

Example :

$$Pr(\text{below } 15) = 0.2$$

Random sample of 3

Jt. prob. or likelihood of 2 under 15 &
1 at least 15 .

$$L = 0.2 \times 0.2 \times 0.8$$

Example :

$P_{insured}$: prob. of insured

Random sample of 3 $\longrightarrow$ 2 insured
1 not "

Jt. prob. or likelihood of observing this :

$$L = P_{ins.} \times P_{ins.} \times (1 - P_{ins.})$$

MLE $\rightarrow$ finds value of $P_{ins.}$ that maximizes L

$P_{ins.} = 0 \longrightarrow L = 0$
$= 0.5 \longrightarrow L = 0.125$
$= 0.7 \longrightarrow L = 0.147$