For each obs. i

$$y_i = \hat{y}_i + \hat{u}_i$$

Total variation (sum of squares)

$$\sum_i (y_i - \bar{y})^2$$

SST

explained variation

SSE

$$\sum_i (\hat{y}_i - \bar{y})^2$$

residual variation

SSR

$$\sum_i \hat{u}_i^2$$

$$SST = SSE + SSR$$

$$R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST}$$

$$0 \le R^2 \le 1$$

Also square of correl^{n} b/w y and $\hat{y}$.

High R^2 : not the ultimate objective!

Functional form

Data : hprice 1

dep. var. (y) $\rightarrow$ price of house (\$1000)

indep. var. (x) $\rightarrow$ size in sq. ft.

① $y = \beta_0 + \beta_1 x + u$

$\hat{\beta}_1 = 0.14$

For $\Delta x = 1$

$\Delta y = 0.14$ (i.e., $\hat{\beta}_1$)

②

$\log(y) = \beta_0 + \beta_1 x + u$

$\hat{\beta}_1 = 0.0004$

Approx. effect of $\Delta x = 1$ $\rightarrow$ $\% \Delta y = 100 \hat{\beta}_1$

$= 0.04\%$

Exact " " $\rightarrow$ $\% \Delta y$

$= 100[\exp(\hat{\beta}_1) - 1]$

$= 0.04\%$

Similar effect for small $\hat{\beta}_1$.

Why logs? constant % effect may be more credible.

Unit - free effects.

③

$\log(y) = \beta_0 + \beta_1 \log(x) + u$

$\hat{\beta}_1 = 0.873$

For $\Delta x = 1\%$, $\Delta y = 0.873\%$ (i.e. $\hat{\beta}_1 \%$)

Expected value of OLS estimators

Under certain assumptions OLS $\rightarrow$ unbiased

$$E(\hat{\beta}) = \beta$$

Assumptions : $\rightarrow$ ① linear model (in parameters; in β's)

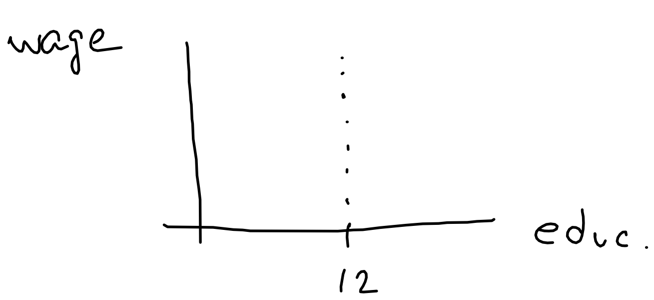
$$y = \beta_0 + \beta_1 x^2 + u$$

$$\log(y) = \beta_0 + \beta_1 \log(x) + u$$

$$y = \beta_0 + \sqrt{\beta_1}\, x + u \quad \times$$

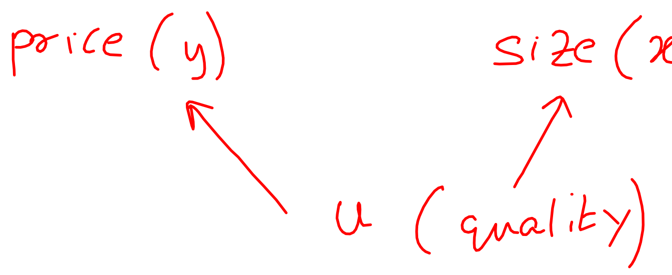
$\rightarrow$ ② random sample e.g. not just price > \$1 mill

$\rightarrow$ ③ Variation in x



$\rightarrow$ ④ $E(u|x) = 0 \rightarrow x$ is exogenous

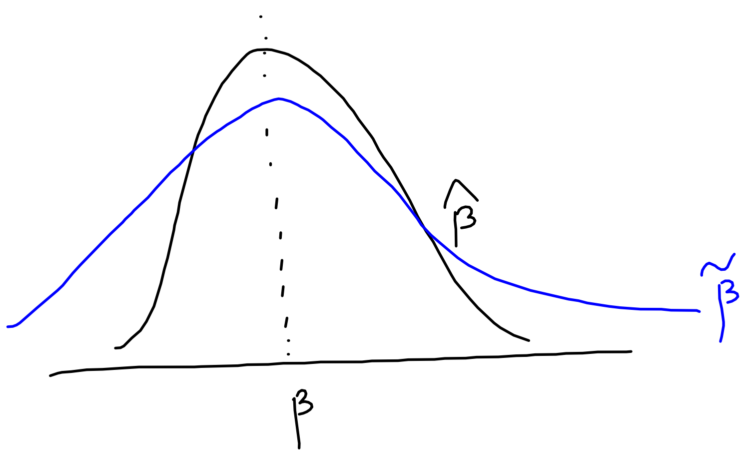
x is not endogenous



$$y = \beta_0 + \beta_1 x + u$$

$$E(y|x) = \beta_0 + \beta_1 x$$

Variance of OLS estimators



variance : precision

$$\text{var}(\hat{\beta}) < \text{var}(\tilde{\beta})$$

Assn. of homoskedasticity

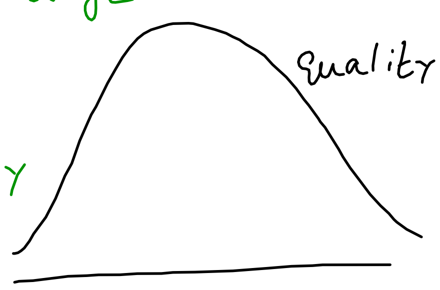
$$\text{Var}(u|x) = \text{Var}(u) = \sigma^2$$

$$\text{var}(y|x) = \sigma^2$$

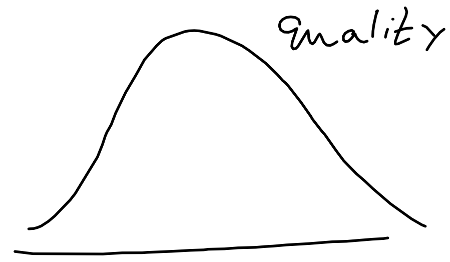
y: price of house ; wage

x: size ; educ.

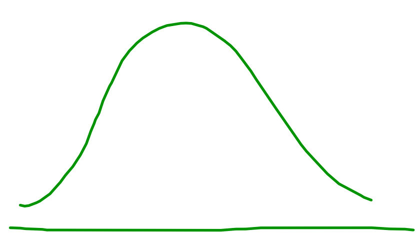
u: quality ; ability



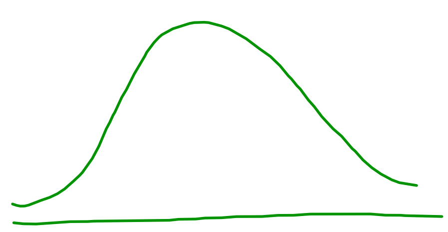
1000 sq. ft.



500 sq. ft.



educ. = 12



educ. = 16