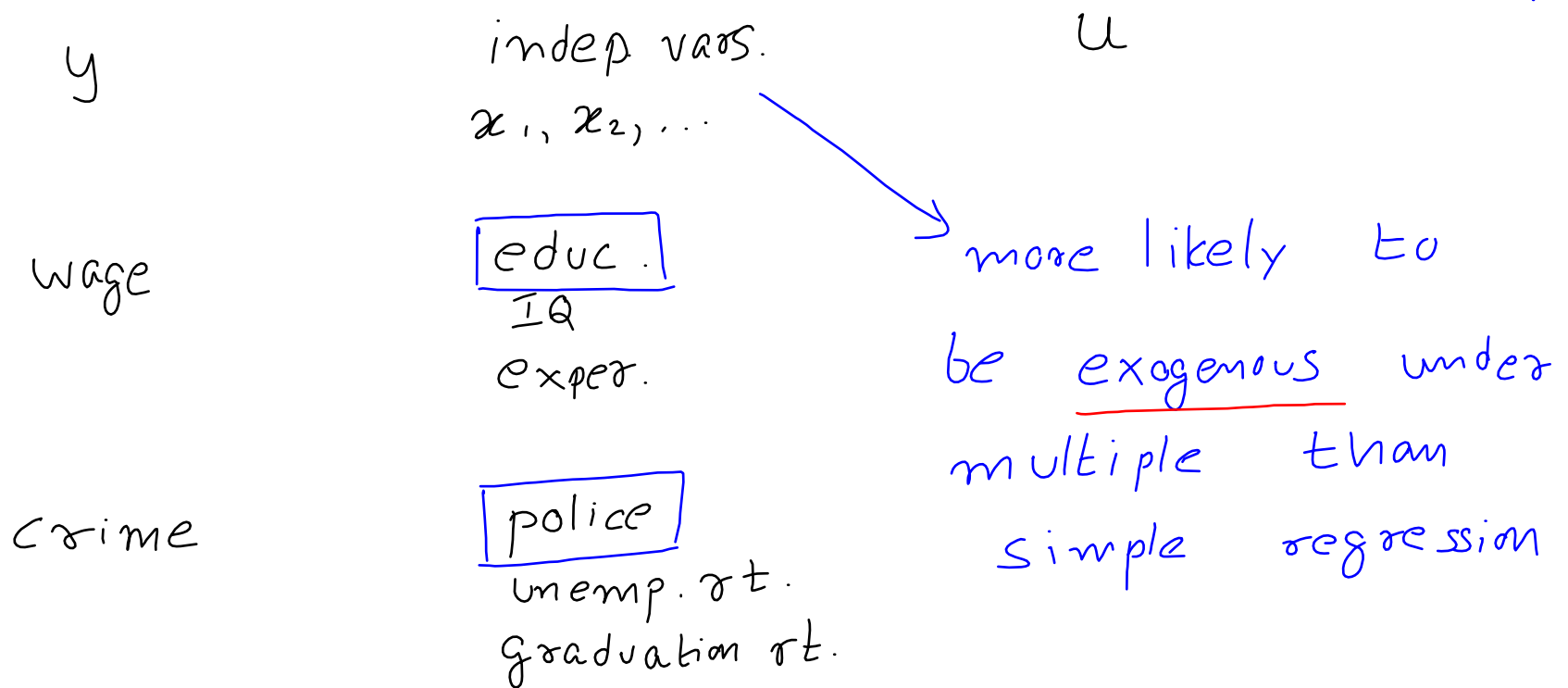


Ch: 3 → Multiple Regression

Motivation:



k indep. vars. k = 2

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

$$\beta_1 : \Delta y \text{ for } \Delta x_1 = 1 ; \quad \begin{array}{l} \Delta x_2 = 0 \\ \Delta u = 0 \end{array}$$

Estimation

$$\text{Assns: } E(u) = 0$$

$$E(x_1 u) = 0$$

$$E(x_2 u) = 0$$

OLS estimates of β_0 , β_1 , and β_2 : $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$

$$\frac{1}{n} \sum_{i=1}^n (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \hat{\beta}_2 x_{i2}) = 0$$

$$\frac{1}{n} \sum_{i=1}^n x_{i1} \square = 0$$

$$\frac{1}{n} \sum_{i=1}^n x_{i2} \square = 0$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2}$$

(fitted value)

$$\hat{u}_i = y_i - \hat{y}_i$$

(residual)

Goodness of fit : $R^2 = \frac{SSE}{SST}$

$= 1 - \frac{SSR}{SST}$ non \downarrow in k (# x 's)

Adjusted R^2 : $\bar{R}^2 = 1 - \frac{\frac{SSR}{(n-k-1)}}{\frac{SST}{(n-1)}}$

As $k \uparrow$ $SSR \downarrow$ $(n-k-1) \downarrow$

n : sample size

Expected value

unbiased
OLS

estimators

$$E(\hat{\beta}_j) = \beta_j$$

$$j = 0, 1, \dots, k$$

↓
x 's

under certain assumptions

linear
in
param.

random
sampling

variation in
each x

no linear
rel? among
regressors

u unrelated to
each x

$$E(u | x_1, x_2, \dots, x_k)$$

$$= E(u)$$

$$= 0$$

$$\text{wage} = \beta_0 + \beta_1 \text{age} + \beta_2 \text{exper} + \beta_3 \text{educ} + u$$

$$\text{age} = 6 + \text{educ} + \text{exper}.$$

Omitted Variable Bias

True model satisfying cond^{ns} for unbiasedness :

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

wage
bought

educ.
smoking

IQ
alcohol

x_2 : omitted ; estimate :

$$y = \beta_0 + \beta_1 x_1 + v$$