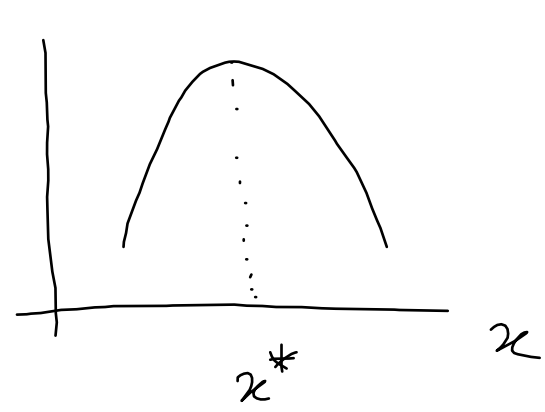
Ch. 6 Quadratic Functions

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$$

$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x$$

$\hat{\beta}_1 > 0$, $\hat{\beta}_2 < 0$



Turning pt. (max.)

at $x^* = \left| \frac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$

BWGHT2

$$bwght = \beta_0 + \beta_1 mage + \beta_2 mage^2 + u$$

$$\hat{\beta}_1 = 97.36 \qquad \hat{\beta}_2 = -1.57$$

$$\frac{\Delta bwght}{\Delta mage} = \beta_1 + 2\beta_2 mage$$

at mage = 25 : $97.36 + 2(-1.57)25 = 18.86$

" = 40 : " 40 = -28.24

$$mage^* = \left| \frac{97.36}{2(-1.57)} \right| = 31.03 \text{ yrs.}$$

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. reg bwght mage magesq

      Source |       SS           df       MS      Number of obs   =     1,832
-------------+----------------------------------   F(2, 1829)      =      7.71
       Model |  5089853.55         2  2544926.78   Prob > F        =    0.0005
    Residual |   603540921     1,829  329984.101   R-squared       =    0.0084
-------------+----------------------------------   Adj R-squared   =    0.0073
       Total |   608630775     1,831  332403.481   Root MSE        =    574.44

------------------------------------------------------------------------------
       bwght | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       mage |    97.35971    25.7041     3.79   0.000     46.94724    147.7722
     magesq |   -1.568884    .429678    -3.65   0.000    -2.411595   -.7261728
      _cons |    1929.752   379.4908     5.09   0.000     1185.471    2674.033
------------------------------------------------------------------------------

. di "Turning point = " abs(_b[mage]/(2*_b[magesq]))
Turning point = 31.028334
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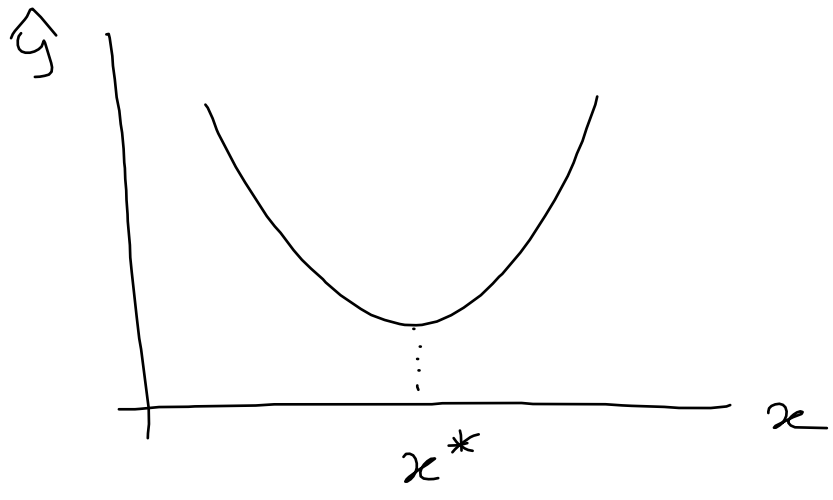
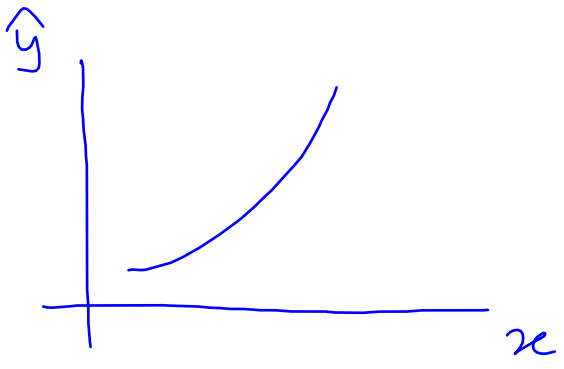
$\hat{\beta}_1 < 0$ & $\hat{\beta}_2 > 0$



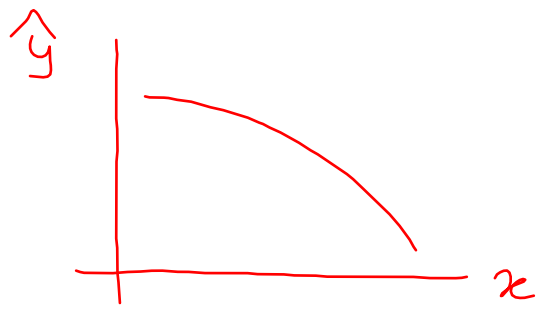
Turning pt. (min.)

at $x^* = \left| \frac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$

$\hat{\beta}_1 > 0$ & $\hat{\beta}_2 > 0$



$\hat{\beta}_1 < 0$ & $\hat{\beta}_2 < 0$



Interaction Terms

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \cdot x_2 + u$$

$$\frac{\Delta y}{\Delta x_1} = \beta_1 + \beta_3 x_2$$

e.g. $\frac{\Delta y}{\Delta x_1}$ at $x_2 = \bar{x}_2$

$$\hat{\beta}_1 + \hat{\beta}_3 \bar{x}_2$$

$$\frac{\Delta y}{\Delta x_2} = \beta_2 + \beta_3 x_1$$

$$\text{bwght} = \beta_0 + \beta_1 \text{mage} + \beta_2 \text{npvis} + \beta_3 \text{mage} \times \text{npvis} + u$$

$$\hat{\beta}_1 = 28.65 \qquad \hat{\beta}_2 = 78.98 \qquad \hat{\beta}_3 = -2.2$$

$\frac{\Delta \text{bwght}}{\Delta \text{mage}} = \beta_1 + \beta_3 \text{npvis}$ → avg. = 11.62

at $\overline{\text{npvis}}$: $28.65 - 2.2 \times 11.62 = 3.086$