Expected value of OLS estimators

Under certain assumptions $\longrightarrow$ OLS unbiased

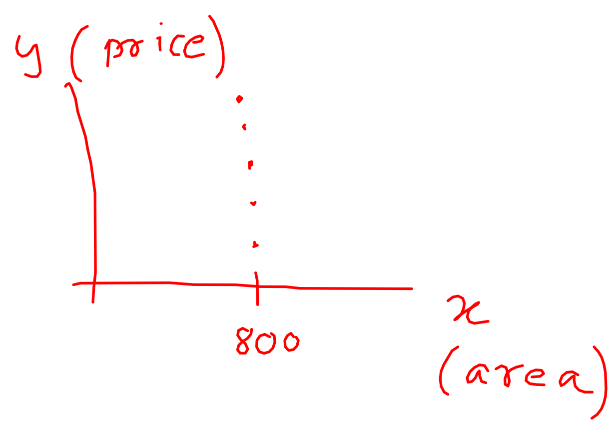
$$E(\hat{\beta}) = \beta$$

Assumptions :

linear model (in parameters i.e. β's)

random sample

variation in x

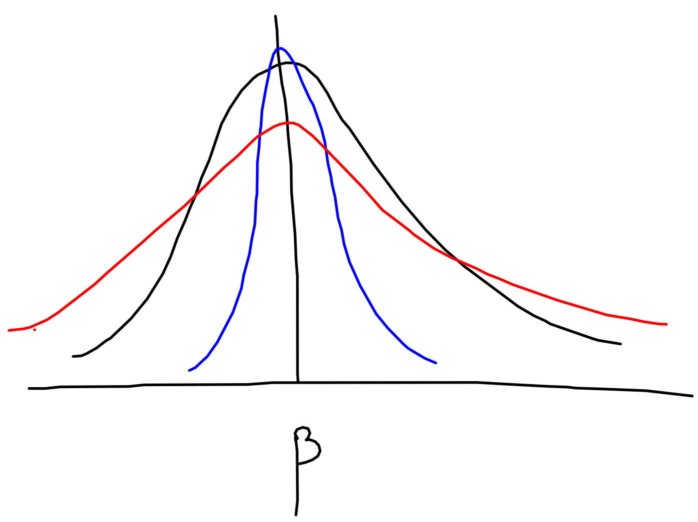


$$E(u|x) = E(u)$$
$$= 0 \longrightarrow x \text{ exogenous}$$

$$E(u|x) \neq 0 \longrightarrow x \text{ endogenous}$$

$$y = \beta_0 + \beta_1 x + u$$

$$E(y|x) = \beta_0 + \beta_1 x$$


β

Variance of OLS estimators

Variance $\longrightarrow$ precision

Assumption of homoskedasticity

$$Var(u|x) = Var(u) = \sigma^2$$

$$y = \beta_0 + \beta_1 x + u$$

$$Var(y|x) = \sigma^2$$

$$\text{var}(\hat{\beta}_1) = \frac{\sigma^2}{\text{variation in } x} = \frac{\sigma^2}{SST_x} = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\text{sd}(\hat{\beta}_1) = \frac{\sigma}{\sqrt{SST_x}}$$

σ : unknown

unbiased estimator of σ^2 : $\hat{\sigma}^2$

$$\hat{\sigma}^2 = \frac{SSR}{(n-2)} = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{(n-2)}$$

2 β's estimated

std. error of $\hat{\beta}_1$: $se(\hat{\beta}_1) = \frac{\hat{\sigma}}{\sqrt{SST_x}}$

$\hat{\sigma}$: standard error of regression

Final thoughts

- unlikely that $\hat{\beta}_1$ from simple regression captures true effect of x on y
- need u and x uncorrelated
- randomized trials help in some cases