## Variance

$k = 3$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

$y$ : price

$x_1$ : sq. ft.

$x_2$ : bdrms

$x_3$ : lot size

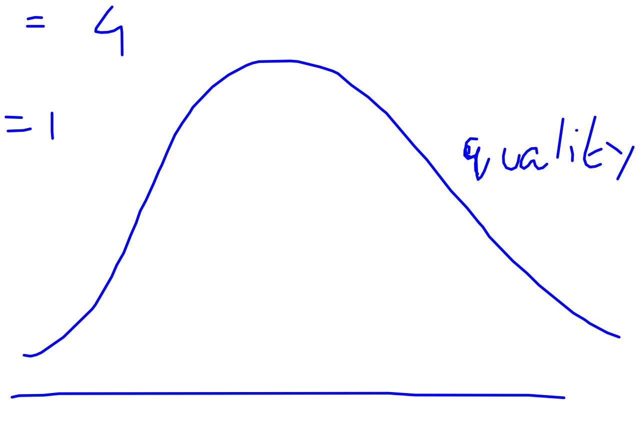
$u$ : quality

Homoskedasticity : $\mathrm{Var}(u \mid x_1, x_2, x_3) = \sigma^2$

$x_1 = 2000$

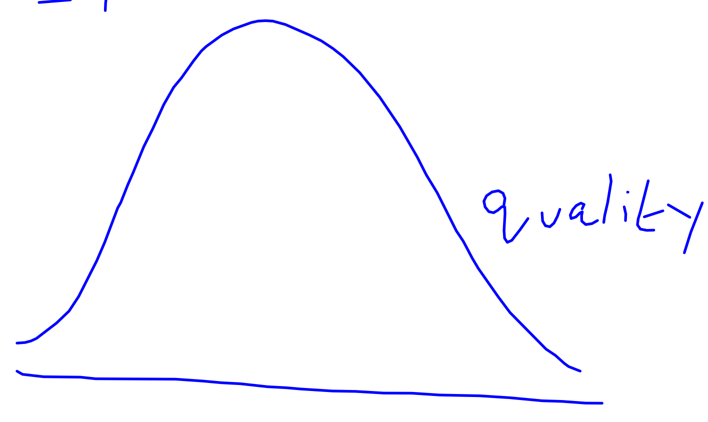
$x_2 = 4$

$x_3 = 1$




$x_1 = 1000$ $\quad x_3 = 0.5$

$x_2 = 1$



$$\mathrm{var}(\hat{\beta}_j) = \frac{\sigma^2}{SST_j(1 - R_j^2)} \qquad j = 1, 2, 3$$

$$SST_j = \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2$$

$R_j^2$ : $R^2$ from reg. of $x_j$ on other $x$'s

$y$ : price

$x_1$ : sq.ft.

$x_2$ : bdrms

$x_3$ : lot size

$$\mathrm{var}(\hat{\beta}_1) = \frac{\sigma^2}{\text{Total variation in sq.ft} \left(1 - R^2 \text{ from reg. of sq.ft. on bdrms \& lot size}\right)}$$

$R_j^2$ close to 1 $\longrightarrow$ multicollinearity

does not violate assn.s reqd. for unbiasedness

Under the assumptions up to homosk.

$\hat{\sigma}^2 \longrightarrow$ unbiased estimator of $\sigma^2$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-k-1} = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-3-1}$$

$k \hookrightarrow$ # of regressors $\quad (k=3)$

$$se(\hat{\beta}_j) = \frac{\hat{\sigma}}{\sqrt{SST_j(1-R_j^2)}} \qquad j = 1, 2, 3$$

# Ch: 4 Inference

$\beta \rightarrow$ unknown

test e.g. $\beta = 0$

using $\hat{\beta}$

3 ways of testing

- is $\hat{\beta}$ too far from hypothesized $\beta$
- how improbable is $\hat{\beta}$
- is hypoth. $\beta$ in $\hat{\beta} - E, \hat{\beta} + E$

# Sampling Distributions

$\hat{\beta}_j = \beta_j +$ a term linear in $u$

$\downarrow$

its distribution follows from that of $u$

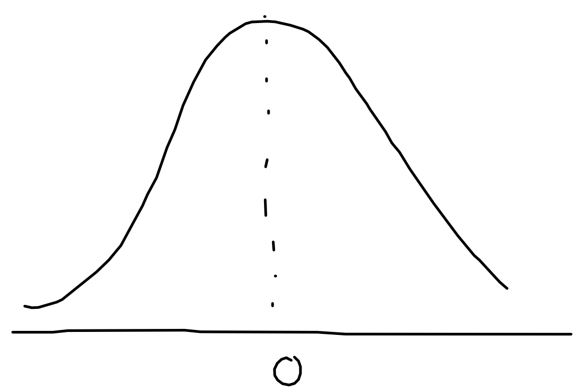
Assⁿ: $u$ indep. of $x_1, \dots, x_k$

↳ normal

$E(u) = 0$

$Var(u) = \sigma^2$

$u \sim N(0, \sigma^2)$



Given all assⁿs up to normality

$\hat{\beta}_j \sim N\left(\beta_j, var(\hat{\beta}_j)\right)$

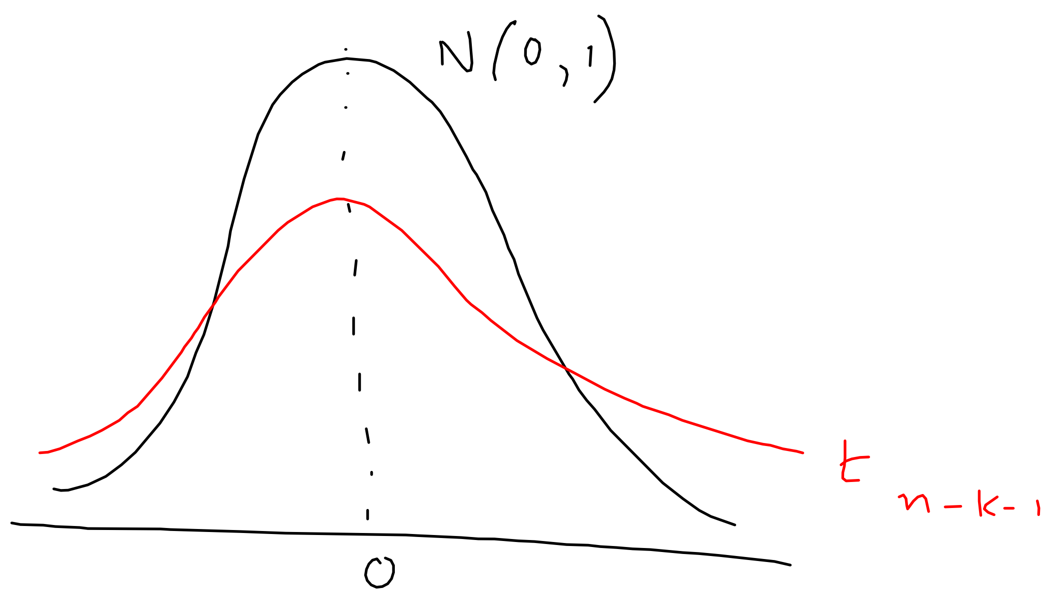
↳ derived in Ch. 3

$$\frac{\hat{\beta}_j - \beta_j}{sd(\hat{\beta}_j)} \sim N(0,1)$$

$$\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim t_{n-k-1}$$ ( $t$ dist. with deg. of freedom, $df = n-k-1$ )

using $\hat{\sigma}$ in place of $\sigma$

$\to N(0,1)$ as $df \to \infty$



N(0,1)
t n-k-1
0