Ch: 3 Multiple Regression

y — indep. vars. $x_1, x_2, \ldots$ — u more likely to satisfy exogeneity

wage — educ. IQ exper.

crime — police unemp. graduation

k indep. vars. e.g. $k = 2$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

β_1 : Δy for $\Delta x_1 = 1$ $\quad \Delta x_2 = 0$

$\Delta u = 0$

Estimation

Assn's :

$$E(u) = 0$$

$$E(x_1 u) = 0$$

$$E(x_2 u) = 0$$

OLS estimates of $\beta_0, \beta_1,$ and β_2 $\longrightarrow$ $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$

$$\frac{1}{n} \sum_{i=1}^{n} \boxed{\left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{i1} - \hat{\beta}_2 x_{i2} \right)} = 0$$

$$\frac{1}{n} \sum_{i=1}^{n} x_{i1} \boxed{} = 0$$

$$\frac{1}{n} \sum_{i=1}^{n} x_{i2} \boxed{} = 0$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} \quad \text{(fitted value)}$$

$$\hat{u}_i = y_i - \hat{y}_i$$

Goodness of fit: $R^2 = \dfrac{SSE}{SST}$

$$= 1 - \frac{SSR}{SST}$$

non $\downarrow$ in k (# x's)

Adjusted R^2 : $\bar{R}^2 = 1 - \dfrac{\frac{SSR}{(n-k-1)}}{\frac{SST}{(n-1)}}$

As $k \uparrow$ $SSR \downarrow$ $(n-k-1) \downarrow$

Expected value

unbiased OLS estimators

$$E(\hat{\beta}_j) = \beta_j \quad j = 0, 1, \dots, k$$

$(\# \, x\text{'s})$

under certain assumptions

linear in parameters

random sampling

Variation in each x

No linear rel^{n}ship among regressors.

$wage = \beta_0 + \beta_1 \, age + \beta_2 \, exper + \beta_3 \, educ + \dots + u$

$age = 6 + educ + exper$

$n \geq k + 1$

u unrelated to each x

$E(u \mid x_1, \dots, x_k) = E(u) = 0$

Omitted Variable Bias

True model satisfying condⁿs for unbiasedness :

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

wage educ. IQ

bwght smoking alcohol

x_2 : omitted

estimate : $y = \beta_0 + \beta_1 x_1 + u$

obtain : $\tilde{\beta}_0$ and $\tilde{\beta}_1$ $\longrightarrow$ biased

$$E(\tilde{\beta}_j) \neq \beta_j \qquad j = 0, 1$$

Bias depends on β_2 and corr. b/w x_2 (omitted) and x_1 (included).