**ECO 5720**
**Problem Set (Practice)**

1. The following equation was obtained by OLS:

$$\widehat{rdintens} = \underset{(0.429)}{2.613} + \underset{(0.00014)}{0.00030}sales - \underset{(0.0000000037)}{0.0000000070}sales^2$$
$$n = 32, \quad R^2 = 0.1484.$$

Here *sales* denotes firm-level sales in millions and *rdintens* represents R&D spending as a percent of sales.

At what point does the marginal effect of *sales* on *rdintens* become negative?

Answer: The turning point is given by $|\widehat{\beta_1}/2\widehat{\beta_2}|$ or 0.0003/(0.000000014) = 21,428.57 (millions of dollars).

2. The following model allows the return to education to depend upon the total amount of both parents' education, called *pareduc*:

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 educ \cdot pareduc + \beta_3 exper + \beta_4 tenure + u.$$

Here, *wage*: monthly earnings; *educ*: years of education; *pareduc*: total years of parents' education; *exper*: years of work experience; and *tenure*: years with current employer.

(i) What is the return to another year of education in this model? You may use the formula for the approximate effect.

Answer: Here,

$$\Delta \log(wage) / \Delta educ = \beta_1 + \beta_2 pareduc.$$

The approximate effect is given by $100(\beta_1 + \beta_2 pareduc)\%$.

(ii) Suppose the estimated equation is

$$\widehat{\log(wage)} = \underset{(0.13)}{5.65} + \underset{(0.010)}{0.047}educ + \underset{(0.00021)}{0.00078}educ \cdot pareduc + \underset{(0.004)}{0.019}exper + \underset{(0.003)}{0.010}tenure$$
$$n = 722, \quad R^2 = 0.169.$$

Calculate the return to another year of education for *pareduc* = 32 and 24. You may use the formula for the approximate effect.

Answer: An additional year of education increases wages by $100(0.047 + 0.00078 pareduc)\%$. For *pareduc* = 32, this is 7.196%. Similarly, for *pareduc* = 24, the return is 6.572%.

3. (i) Using the data in WAGE1, estimate the equation

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + u$$

and report the results.

Answer:

```
. reg lwage educ exper expersq

      Source |       SS           df       MS      Number of obs   =       526
-------------+----------------------------------   F(3, 522)       =     74.67
       Model |  44.5393713         3  14.8464571   Prob > F        =    0.0000
    Residual |   103.79038       522  .198832146   R-squared       =    0.3003
-------------+----------------------------------   Adj R-squared   =    0.2963
       Total |  148.329751       525   .28253286   Root MSE        =    .44591

------------------------------------------------------------------------------
       lwage | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        educ |   .0903658    .007468    12.10   0.000     .0756948    .1050368
       exper |   .0410089   .0051965     7.89   0.000     .0308002    .0512175
     expersq |  -.0007136   .0001158    -6.16   0.000     -.000941   -.0004861
       _cons |   .1279975   .1059323     1.21   0.227    -.0801085    .3361035
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(ii) Using the approximation $\%\Delta wage = 100(\widehat{\beta_2} + 2\widehat{\beta_3}exper)\Delta exper$, find the approximate return to the fifth year of experience. What is the approximate return to the twentieth year of experience?

Answer: To estimate the return to the fifth year of experience, we set *exper* = 4 and increase *exper* by one. Wage increases by 100[0.0410 -2 (0.000714)4], i.e., 3.53%.

Similarly, for the 20$^{th}$ year of experience, wage increases by 100[0.0410 -2 (0.000714)19], i.e., 1.39%.

(iii) After what value of *exper* does additional experience actually lower predicted log(*wage*)? How many people have more experience in this sample?

Answer: This is given by the turning point which is about 0.041/[2(0.000714)] = 28.71 years of experience. In the sample, there are 121 people with at least 29 years of experience.

4. (i) Using the data in GPA2, estimate the model

$$sat = \beta_0 + \beta_1 hsize + \beta_2 hsize^2 + u.$$

Here, sat denotes the SAT score and hsize is the size of the graduating class (in hundreds).

Answer:

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. reg sat hsize hsizesq
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| Source | SS | df | MS | | | |
|---|---|---|---|---|---|---|
| | | | | Number of obs | = | 4,137 |
| | | | | F(2, 4134) | = | 15.93 |
| Model | 614822.097 | 2 | 307411.048 | Prob > F | = | 0.0000 |
| Residual | 79759024.2 | 4,134 | 19293.4263 | R-squared | = | 0.0076 |
| | | | | Adj R-squared | = | 0.0072 |
| Total | 80373846.3 | 4,136 | 19432.7481 | Root MSE | = | 138.9 |

| sat | Coefficient | Std. err. | t | P>\|t\| | [95% conf. | interval] |
|---|---|---|---|---|---|---|
| hsize | 19.81446 | 3.990666 | 4.97 | 0.000 | 11.99061 | 27.63831 |
| hsizesq | -2.130606 | .549004 | -3.88 | 0.000 | -3.206949 | -1.054263 |
| _cons | 997.9805 | 6.203448 | 160.88 | 0.000 | 985.8184 | 1010.143 |

(ii) Using the estimated equation from part (i), what is the "optimal" high school size? Justify your answer.

Answer: SAT score increases with size of class but there are also diminishing returns to class size. Thus, for high values of class size the overall impact on SAT score may even be negative. Prior to that, at some value of class size, *hsize**, the SAT score reaches its maximum. This is the turning point which is calculated as *hsize** = 19.814/[2(2.131)] = 4.649. Since *hsize* is in 100s, the "optimal" class size is 465.