Ch. 8: Heteroskedasticity

Consequences

$$\mathrm{var}(u \mid x_1 \ldots x_k) \neq \sigma^2$$

but depends on x_j

$$\text{price} = \beta_0 + \beta_1\, \text{lotsize} + \beta_2\, \text{sq.ft.} + \beta_3\, \text{bdrms} + u$$

(quality)

$\mathrm{var}(u \mid \text{lotsize, sq.ft., bdrms})$ depends on sq.ft.

Under ass^ns for unbiasedness of $\hat{\beta}_j$ $\longrightarrow$ unbiased for β_j

usual std. errors & test statistics $\longrightarrow$ no longer valid.

Heteroskedasticity - Robust Inference

Heterosk. of unknown form.

" robust std. errors available in case of large samples.

Simple regression: $se(\hat{\beta}_1) = \dfrac{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \hat{u}_i^2}}{SST_x}$

$$SST_x = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Multiple regression

$$se(\hat{\beta}_j) = \frac{\sqrt{\sum_{i=1}^{n} \hat{r}_{ij}^2 \, \hat{u}_i^2}}{SST_j \, (1 - R_j^2)}$$

$score = \beta_0 + \beta_1 \, classsize$
$+ \beta_2$ PC stud. exp.
$+ \beta_3$ % minority
$+ u$

SST_j: $\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2$

R_j^2: R^2 from reg. of x_j on all other x's

$\hat{r}_{ij}$: residual from reg. of x_j on all other x's for obs. i

$se(\hat{\beta}_1) \rightarrow SST_{cs} \rightarrow$ total variation in class size

$R^2_{cs} \rightarrow R^2$ from reg. of class size on PC stud. exp. & % minority

$\hat{r}_{cs} \rightarrow$ residual from above reg.

Testing

Model (with ass$^{\underline{n}}$s up to those reqd. for unbiased $\hat{\beta}$)

$$y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + u$$