Logit

$u \sim$ logistic distribution

$$p(u \le \beta_0 + \beta_1 x) = \frac{\exp(\beta_0 + \beta_1 x)}{1 + \exp(\beta_0 + \beta_1 x)}$$

$\hat{\beta}_0$, $\hat{\beta}_1$: from MLE

Maximum Likelihood Estimation (MLE)

Example $P(\text{below } 15) = 0.2$

Random sample of 3

Jt. prob. or likelihood of 2 under 15 & 1 at least 15

$$L = 0.2 \times 0.2 \times 0.8$$

Example

$P_{insured}$ = prob. of insured

Random sample of 3 → 2 ins. & 1 not ins.

Jt. prob. or likelihood of observing this

$$L = P_{ins.} \times P_{ins.} \times (1 - P_{ins.})$$

MLE finds $P_{ins.}$ that maximizes L

↳ an estimate of $P_{ins.}$ that maximizes the likelihood of observing the data that we obtain.

$P_{ins.} = 0 \rightarrow L = 0$

$= 0.5 \rightarrow L = 0.125$

$= 0.7 \rightarrow L = 0.147$

Value that max. $L \longrightarrow P_{ins.} = 2/3$.

In case of logit or Probit with

$y = 1$ or 0

$\downarrow$ ins. $\qquad \downarrow$ not ins.

$$P(y=1 \mid x) = G(\beta_0 + \beta_1 x)$$

$$L = G(\cdot) \times G(\cdot) \times [1 - G(\cdot)]$$

MLE : finds $\hat{\beta}_0$ and $\hat{\beta}_1$ that maximizes L or $\log(L)$.

Interpretation

Probit : continuous x

$$\frac{\Delta P(y=1|x)}{\Delta x} = (\text{std. normal density at } x)\ \beta_1$$

Logit : cont. x

$$\frac{\Delta P(y=1|x)}{\Delta x} = (\text{logistic density at } x)\ \beta_1$$

Effects depend on x.

Typically calculate:

- effect at avg. value of x
- avg. of effects across all values of x