Two-Period Panel Data Analysis

Same units observed over 2 time pds.

- Model

$$y_{it} = \beta_0 + \delta_0 d2_t + \beta_1 x_{it} + v_{it}$$

  - i : person, firm, city, etc. and t : time period
  - $d2$: dummy var. for pd. 2 (1 for pd. 2, 0 for pd. 1)

- Example

$$crime_{it} = \beta_0 + \delta_0 d2_t + \beta_1 unem_{it} + v_{it}$$
$$prod_{it} = \beta_0 + \delta_0 d2_t + \beta_1 expo_{it} + v_{it}$$

Intercept for pd. 1 $= \beta_0$

Intercept for pd. 2 $= \beta_0 + \delta_0$

Two-Period Panel Data Analysis (cont.)

- Suppose

$$y_{it} = \beta_0 + \delta_0 d2_t + \beta_1 x_{it} + a_i + u_{it}$$

  - a_i : unobserved effect ; fixed effect ; unobs. heterogen-eity
  - u_{it} : idiosyncratic error
  - v_{it} : time varying error
  - $= a_i + u_{it}$: composite error

- Example

$$crime_{it} = \beta_0 + \delta_0 d2_t + \beta_1 unem_{it} + city_i + u_{it}$$
$$prod_{it} = \beta_0 + \delta_0 d2_t + \beta_1 expo_{it} + mqual_i + u_{it}$$

Two-Period Panel Data Analysis (cont.)

- Estimating β_1

$$y_{it} = \beta_0 + \delta_0 d2_t + \beta_1 x_{it} + a_i + u_{it}$$

- Pooling the two years and performing OLS : may not be unbiased if e.g. a_i and x_{it} are correlated
- One solution: difference the data

Two-Period Panel Data Analysis (cont.)

- Two years

$$y_{i2} = (\beta_0 + \delta_0) + \beta_1 x_{i2} + a_i + u_{i2}$$
$$y_{i1} = \beta_0 + \beta_1 x_{i1} + a_i + u_{i1}$$

- Subtracting

$$y_{i2} - y_{i1} = \delta_0 + \beta_1 (x_{i2} - x_{i1}) + u_{i2} - u_{i1}$$

- The *first-differenced equation*

$$\Delta y_i = \delta_0 + \beta_1 \Delta x_i + \Delta u_i$$

β_1 : first-differenced estimator

- Example

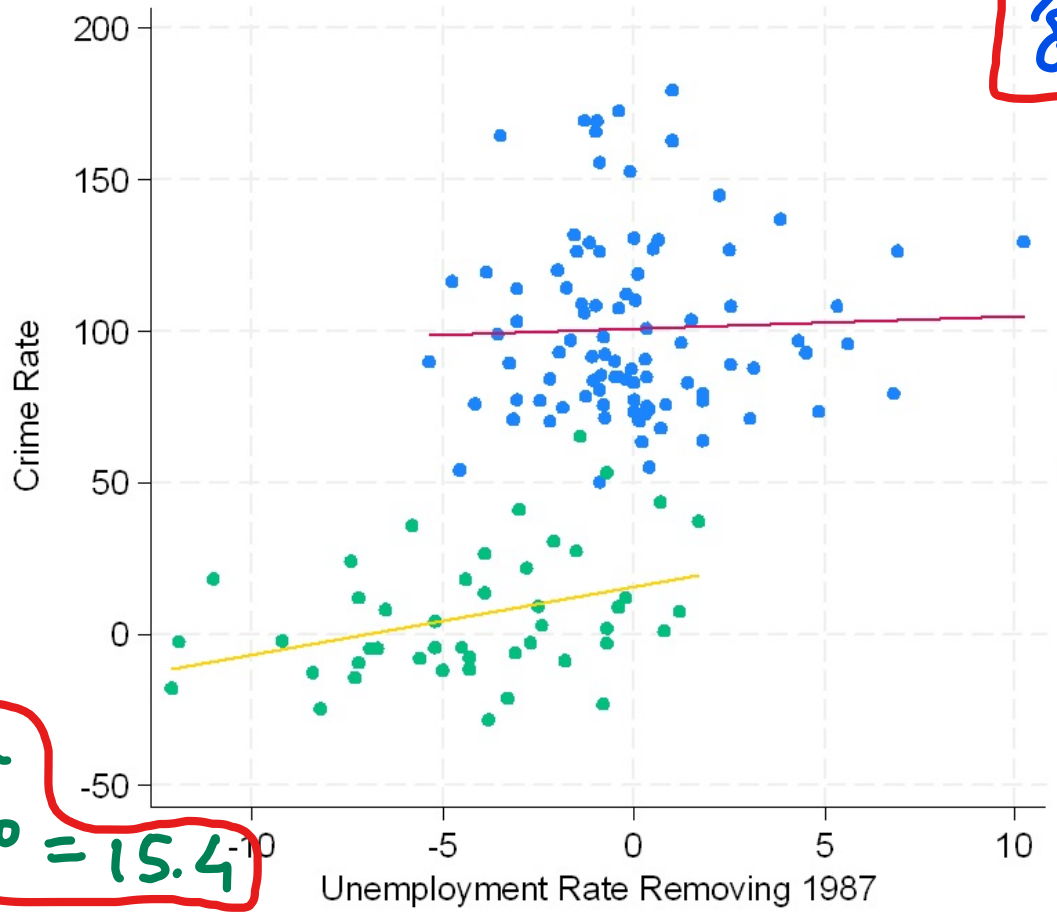
$$\Delta crime_i = \delta_0 + \beta_1 \Delta unem_i + \Delta u_i$$
$$\Delta prod_i = \delta_0 + \beta_1 \Delta expo_i + \Delta u_i$$

Two-Period Panel Data Analysis (cont.)

$$crmrte_{it} = \beta_0 + \delta_0 d87_t + \beta_1 unem_{it}$$

$\hat{\delta}_0 = 7.94$
$\hat{\beta}_1 = 0.427$

$+ a_i + u_{it}$



$\hat{\delta}_0 = 15.4$

$\Delta crmrte_i = \delta_0 + \beta_1 \Delta unem_i + \Delta u_i$

$\hat{\beta}_1 = 2.22$

a_i: unobs. city effect → industry composition, geography, etc.

u_{it}: idiosyncratic errors e.g. weather shocks, protests/activism

Two-Period Panel Data Analysis (cont.)

$(u_{i2} - u_{i1})$ should be uncorrelated with $(x_{i2} - x_{i1})$; u_i should be uncorr. with x_i from both time pds.

- Note
  - Still need Δu_i to be uncorrelated with Δx_i
  - The *strict exogeneity* assumption
  - Need variation in Δx_i

u & x uncorr. across all time pds. → strict exogeneity

x: strictly exogenous

u and x are uncorr. in same time pd. ⇒ contemporaneous exogeneity