

Additional Panel Data Methods

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- ① Fixed Effects Estimation
- ② Random Effects Models

Fixed Effects Estimation

i : indiv. units

t : 1, 2, ..., T

$$\bar{y}_i = \frac{1}{T} \sum_{t=1}^T y_{it}$$

- Alternative to first differencing to eliminate time-inv. unobs. a_i
- Model

$$y_{it} = \beta_0 + \beta_1 x_{it} + a_i + u_{it}$$

- Averaging over each i

$$\bar{y}_i = \beta_0 + \beta_1 \bar{x}_i + a_i + \bar{u}_i$$

- Subtracting

$$y_{it} - \bar{y}_i = \beta_1 (x_{it} - \bar{x}_i) + u_{it} - \bar{u}_i$$

$$\ddot{y}_{it} = \beta_1 \ddot{x}_{it} + \ddot{u}_{it}$$

↓
estimate by OLS

under strict exog. → unbiasedness

Fixed Effects Estimation (cont.)

- Example

$$crime_{it} = \beta_0 + \beta_1 unem_{it} + city_i + u_{it}$$

- Averaging over time

$$\overline{crime}_i = \beta_0 + \beta_1 \overline{unem}_i + city_i + \bar{u}_i$$

- Subtracting

$$crime_{it} - \overline{crime}_i = \beta_1 (unem_{it} - \overline{unem}_i) + u_{it} - \bar{u}_i$$

$$c\ddot{r}i\ddot{m}e_{it} = \beta_1 un\ddot{e}m_{it} + \ddot{u}_{it}$$

Fixed Effects Estimation (cont.)

- Note (cont.)
 - ▶ Dummy variable regression

$$y_{it} = \beta_0 + \beta_1 x_{it} + a_i + u_{it}$$

- ★ Include a dummy variable for *each cross-sectional unit (i.e. i)*
- ★ Results in the same estimates of *instead of within transferⁿ*

↙
 β_1 as from FEs
reg. on time-demanded
data

Fixed Effects Estimation (cont.)

- Note (cont.)

- ▶ Fixed effects versus first-differenced estimators

- ★ For $T = 2$, both are identical

- ★ For $T \geq 3$, the two are not identical, but both

- ★ Inference (standard errors)

consistent
under ass^{ns}
of strict
exog.

↙
based on ass^{ns}
related to u_{it}
versus Δu_{it}

Fixed Effects Estimation (cont.)

- Note (cont.)

- ▶ Balanced versus unbalanced panel

- ★ Unbalanced:

- ★ Balanced:

- ▶ Unbalanced panel:

all units not
observed for all pds.

all units observed
for all pds.

↓
unbiased/consistent
estimates provided
reason for missing data
not correl. with U_{it}

Random Effects Models

- Model

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \dots + \beta_k x_{itk} + a_i + u_{it}$$

- a_i assumed to be *uncorr. w/ x_{itj}*
- Composite error: $v_{it} = a_i + u_{it}$ - *correlated over time*
- Estimation via feasible generalized least squares (FGLS)
- Note: allows *for coeffs. of time-inv. x 's to be estimated*