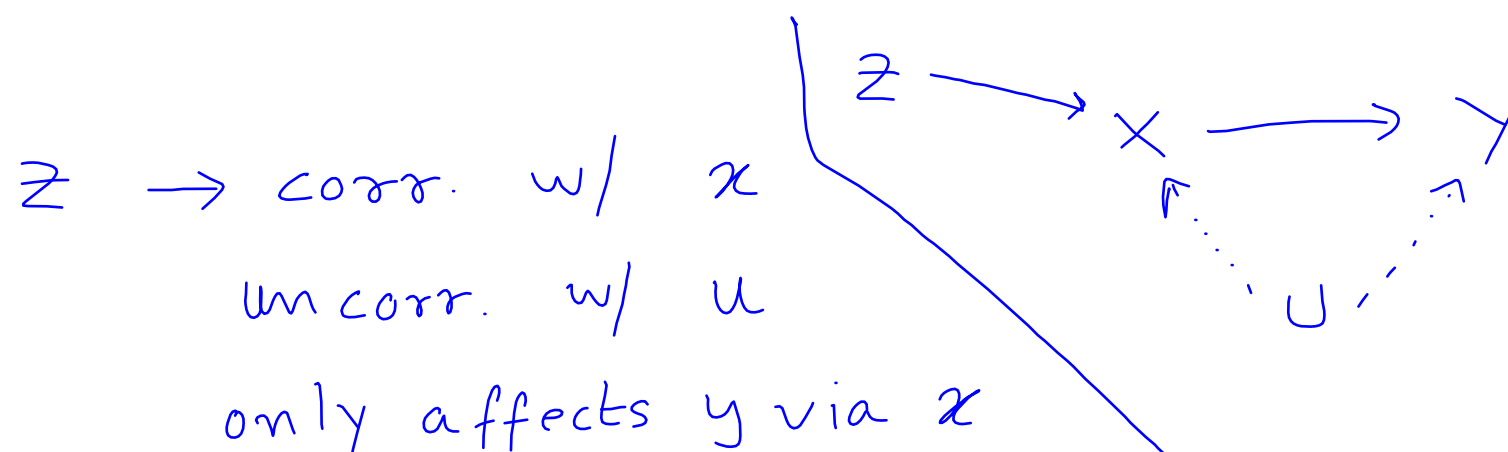


Ch. 15 : Instrumental Variables

$$y = \beta_0 + \beta_1 x + u$$

↓
endog. → OLS estimator biased



y	x	z
grade	attendance	dist. from school
grade	charter school	lottery
lab. ss	# child	BB, GG

Possible to estimate β_1 consistently.

IV estimator $\rightarrow$ consistent
not unbiased

Need: ① z & u uncorr. $\rightarrow$ instrument exogeneity
difficult to test

② z & x corr. $\rightarrow$ instrument relevance
can be tested by
regressing x on z

① & ② $\Rightarrow$ IV estimator consistent (not unbiased)

$$y = \beta_0 + \beta_1 x + u$$

$\downarrow$
 endog.

$z \rightarrow$ instrument

$$\text{IV estimator of } \beta_1 \text{ (} \hat{\beta}_1 \text{)} = \frac{\sum_i (z_i - \bar{z})(y_i - \bar{y})}{\sum_i (z_i - \bar{z})(x_i - \bar{x})}$$

Can also be estimated from:

$$E(u) = 0$$

$$E(z \cdot u) = 0$$

$$\text{Homoskedasticity} \Rightarrow E(u^2 | z) = \sigma^2 = \text{Var}(u)$$

Asymptotic std. error of $\hat{\beta}_1$:

$$\sqrt{\frac{\hat{\sigma}^2}{SST_x R^2_{x,z}}}$$

$\hookrightarrow R^2$ from reg. of x on z

$$\hat{\sigma}^2 = \frac{\sum_i \hat{u}_i^2}{n-2} \quad \hat{u}_i : \text{IV residuals.}$$

$R^2_{x,z} < 1 \Rightarrow \text{IV std. error} > \text{OLS std. error}$

$\hookrightarrow$ small if x and z are slightly correl.
 $\Rightarrow$ large std. errors

If $\text{corr.}(x, z)$ is low

$\Rightarrow$ problem of weak
instruments

Inconsistency in IV estimator related

$\frac{\text{corr.}(z, u)}{\text{corr.}(z, x)} \Rightarrow$ asymptotic bias
can be large even if
 $\text{corr.}(z, u)$ is small

y	x	z
$\log(\text{wage})$	educ.	father's educ.

$$\hat{\beta}_{OLS} = 0.109$$

$$\hat{\beta}_{IV} = 0.059 = \frac{\text{cov}(z, y)}{\text{cov}(z, x)}$$

2 stage least squares

1st stage $\rightarrow$ regress x on z
 obtain predicted values $\rightarrow \hat{x}$

2nd stage $\rightarrow$ regress y on $\hat{x}$
 $\hookrightarrow$ gives $\hat{\beta}_{IV}$