

Multiple Regression Analysis

- ① Motivation
- ② Estimation
- ③ Expected Value
- ④ Variance

Motivation

Estimation

Example

y (wage)	x_1 (educ)	x_2 (exper)
3.1	11	2
3.2	12	22
3	11	2
6	8	44
5.3	12	7
8.8	16	9
11	18	15
5	12	5
3.6	12	26
18	17	22

Estimation (cont.)

$$\bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}_1 - \hat{\beta}_2 \bar{x}_2 = 0$$

$$\overline{x_1 y} - \hat{\beta}_0 \bar{x}_1 - \hat{\beta}_1 \overline{(x_1)^2} - \hat{\beta}_2 \overline{x_1 x_2} = 0$$

$$\overline{x_2 y} - \hat{\beta}_0 \bar{x}_2 - \hat{\beta}_1 \overline{x_1 x_2} - \hat{\beta}_2 \overline{(x_2)^2} = 0$$

$$6.742 - \hat{\beta}_0 - 12.9\hat{\beta}_1 - 15.4\hat{\beta}_2 = 0$$

$$97.234 - 12.9\hat{\beta}_0 - 175.1\hat{\beta}_1 - 190.4\hat{\beta}_2 = 0$$

$$115.064 - 15.4\hat{\beta}_0 - 190.4\hat{\beta}_1 - 396.8\hat{\beta}_2 = 0$$

$$\hat{\beta}_0 = -12.317 \quad \hat{\beta}_1 = 1.312 \quad \hat{\beta}_2 = 0.138$$

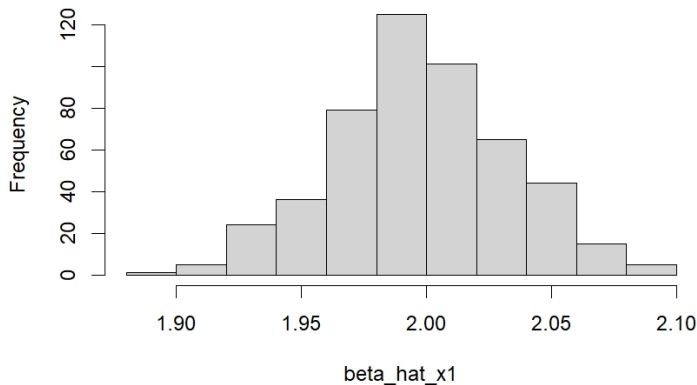
Expected Value

Omitted Variable Bias

Expected Value (cont.)

- $n = 1000$, $\text{reps} = 500$, $\text{corr}(x_1, x_2) = 0.4$, $\text{corr}(x_1, u) = 0$, $\text{corr}(x_2, u) = 0$
- $y = 1 + 2x_1 + x_2 + u$

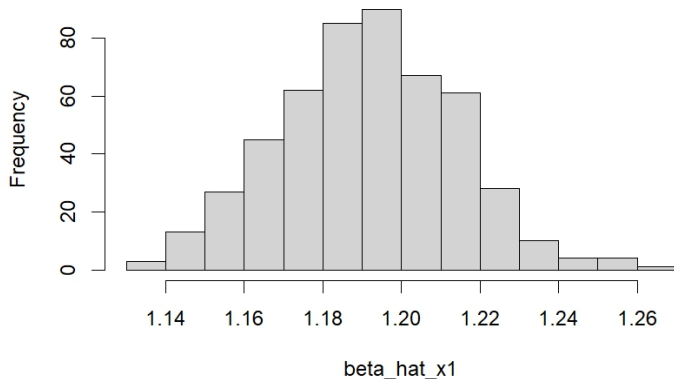
Histogram of beta_hat for x1



Expected Value (cont.)

- $n = 1000$, reps = 500, $\text{corr}(x_1, x_2) = 0.4$, $\text{corr}(x_1, u) = -0.6$, $\text{corr}(x_2, u) = 0.2$
- $y = 1 + 2x_1 + x_2 + u$

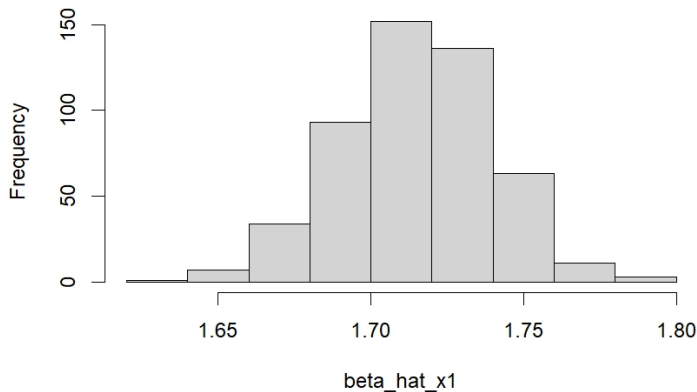
Histogram of beta_hat for x1



Expected Value (cont.)

- $n = 1000$, $\text{reps} = 500$, $\text{corr}(x_1, x_2) = 0.4$, $\text{corr}(x_1, u) = 0$, $\text{corr}(x_2, u) = 0.6$
- $y = 1 + 2x_1 + x_2 + u$

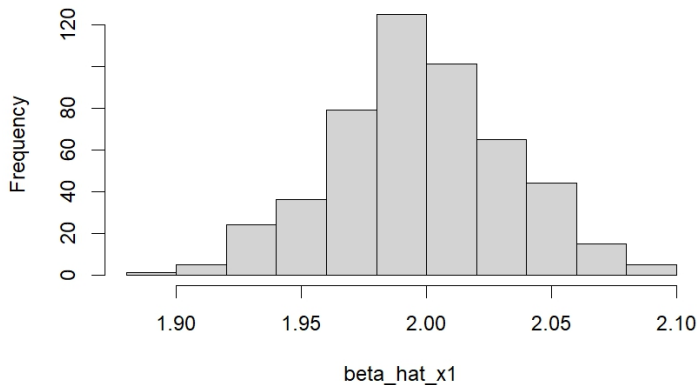
Histogram of beta_hat for x1



Variance

- $n = 1000$, $\text{reps} = 500$, $\text{corr}(x_1, x_2) = 0.4$, $\text{corr}(x_1, u) = 0$, $\text{corr}(x_2, u) = 0$
- $y = 1 + 2x_1 + x_2 + u$

Histogram of beta_hat for x1



Variance (cont.)

- $n = 1000$, $\text{reps} = 500$, $\text{corr}(x_1, x_2) = 0.99$, $\text{corr}(x_1, u) = 0$, $\text{corr}(x_2, u) = 0$
- $y = 1 + 2x_1 + x_2 + u$

Histogram of beta_hat for x1

