

Additional Panel Data Methods

- 1 Fixed Effects Estimation
- 2 Random Effects Models

Fixed Effects Estimation

- Alternative to first differencing *to eliminate time-inv. unobs. a_i*
- Model

$$y_{it} = \beta_0 + \beta_1 x_{it} + a_i + u_{it}$$

i: indiv. units

- Averaging over each i

$$\bar{y}_i = \beta_0 + \beta_1 \bar{x}_i + a_i + \bar{u}_i$$

$t=1, \dots, T$

$$\bar{y}_i = \frac{\sum_{t=1}^T y_{it}}{T}$$

- Subtracting

$$y_{it} - \bar{y}_i = \beta_1 (x_{it} - \bar{x}_i) + u_{it} - \bar{u}_i$$
$$\ddot{y}_{it} = \beta_1 \ddot{x}_{it} + \ddot{u}_{it}$$

Fixed Effects Estimation (cont.)

- Example

$$crime_{it} = \beta_0 + \beta_1 unem_{it} + city_i + u_{it}$$

- Averaging over time

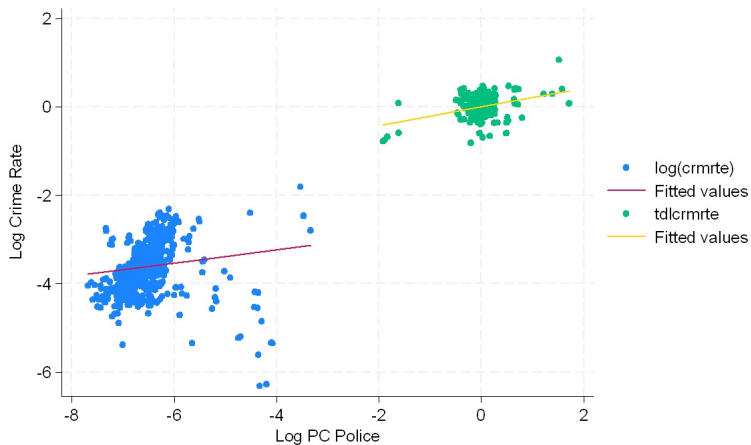
$$\overline{crime}_i = \beta_0 + \beta_1 \overline{unem}_i + city_i + \bar{u}_i$$

- Subtracting

$$crime_{it} - \overline{crime}_i = \beta_1 (unem_{it} - \overline{unem}_i) + u_{it} - \bar{u}_i$$

$$\ddot{crime}_{it} = \beta_1 \ddot{unem}_{it} + \ddot{u}_{it}$$

Fixed Effects Estimation (cont.)



Fixed Effects Estimation (cont.)

- Time-demeaned variables: $\bar{y}_{it}$, $\bar{x}_{it}$, and $\bar{u}_{it}$
 - ▶ Also called the **fixed effects / within transformation**
- OLS estimator based on the time-demeaned variables:

**fixed effects / within
estimator**

**Relies on time variation in y and x
within each cross-sectional unit.**

Fixed Effects Estimation (cont.)

i : obs.

t : time

j : var. e.g. unem., avg sen, etc. ...

arbitrary correlⁿ b/w
 x_{ijt}

• Note

- ▶ Fixed effects estimator allows for
- ▶ Fixed effects transformation eliminates any indep. var. that is
- ▶ Strict exogeneity $\Rightarrow u_{it}$ should be uncorrelated with each x_{ijt} across all time periods

★ Example

$$crime_{it} = \beta_0 + \beta_1 unem_{it} + city_i + u_{it}$$

under strict exog.: fixed effects

estimator

$$wage = \beta_0 + \beta_1 educ + \beta_2 race + abil + u$$

const. over time $\rightarrow$ such factors are controlled for!

Fixed Effects Estimation (cont.)

- Note (cont.)

- ▶ Dummy variable regression

$$y_{it} = \beta_0 + \beta_1 x_{it} + a_i + u_{it}$$

- ★ Include a dummy variable for each cross-sectional unit
- ★ Results in the same estimates of

β_1 as obtained
from regression
on time-demeaned data

(along with
other
explanatory
vars.)

Fixed Effects Estimation (cont.)

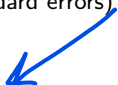
- Note (cont.)

- ▶ Fixed effects versus first-differenced estimators

- ★ For $T = 2$, both are identical

- ★ For $T \geq 3$, the two are not identical, but both *consistent*

- ★ Inference (standard errors)



depends on
assumptions related
to u_{it} & Δu_{it}

consistent
under assumption
of
strict
exog.

Fixed Effects Estimation (cont.)

$\text{Food sec.}_{it} = \dots \beta \text{SNAP}_{it} + u_{it}$
unbiased & consistent estimators provided
reason for missing data not corr. w/
 u_{it}

- Note (cont.)

- ▶ Balanced versus unbalanced panel

- ★ Unbalanced:

- ★ Balanced:

- ▶ Unbalanced panel:

→ certain yrs./time pds.
missing for some
cross-sectional units

all yrs. or pds. of data observed
for all cross-sectional units

Random Effects Models

- Model

$$y_{it} = \beta_0 + \beta_1 x_{it1} + \dots + \beta_k x_{itk} + a_i + u_{it}$$

- a_i assumed to be *uncorr. w/ x_{ijt} across all time pds.*
- Composite error: $v_{it} = a_i + u_{it}$ - *corr. across time*
- Estimation via feasible generalized least squares (FGLS)
- Note: allows *time-invariant explanatory vars.*