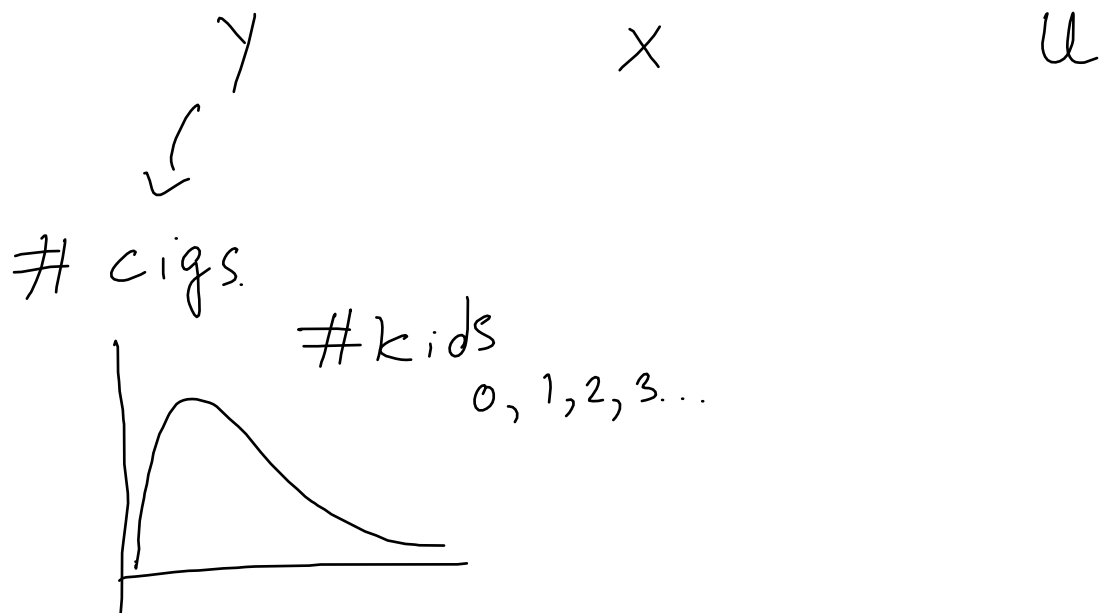


Chapter 5

① Asymptotic normality of $\hat{\beta}$



② Consistency

- no asymptotic bias
- if unbiased then less dispersed in large samples

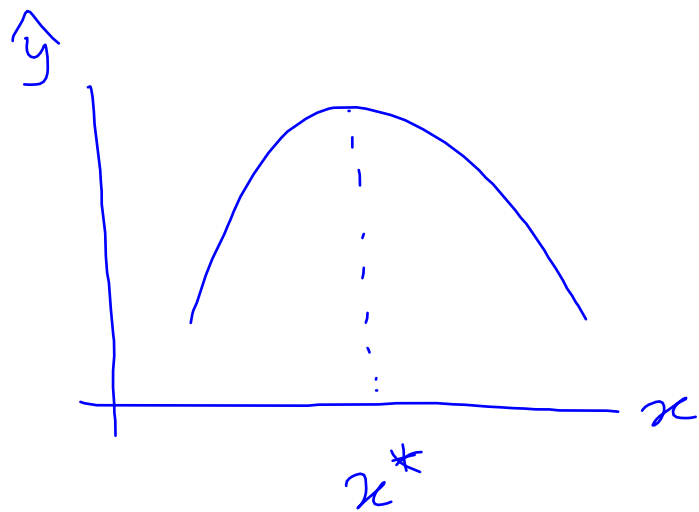
The graph shows two normal distributions centered at β . The narrower distribution is labeled $\hat{\beta}$ (large n) and the wider one is labeled $\hat{\beta}$ (small n).

Ch. 6 Quadratic Functions

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$$

$$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x$$

$$\hat{\beta}_1 > 0 \quad \& \quad \hat{\beta}_2 < 0$$



Turning pt. (max.)
at $x^* = \left| \frac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$

BW GHT2

$$\text{bwght} = \beta_0 + \beta_1 \text{mage} + \beta_2 \text{mage}^2 + u$$

$$\hat{\beta}_1 = 97.36$$

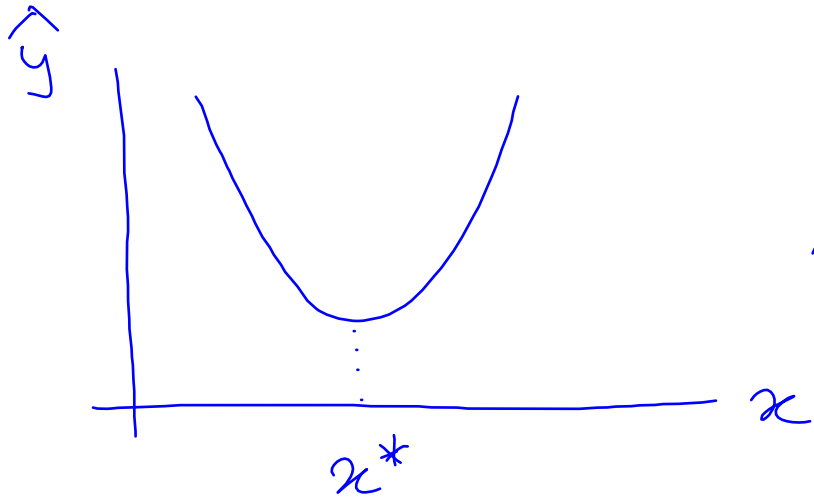
$$\hat{\beta}_2 = -1.57$$

$$\frac{\Delta \text{bwght}}{\Delta \text{mage}} = \beta_1 + 2\beta_2 \text{mage}$$

$$\begin{aligned} \text{at } \text{mage} = 25 &: 97.36 + 2(-1.57)25 = 18.86 \\ \text{" } = 40 &: \text{" } 40 = -28.24 \end{aligned}$$

$$\text{mage}^* = \left| \frac{97.36}{2(-1.57)} \right| = 31.03 \text{ yrs}$$

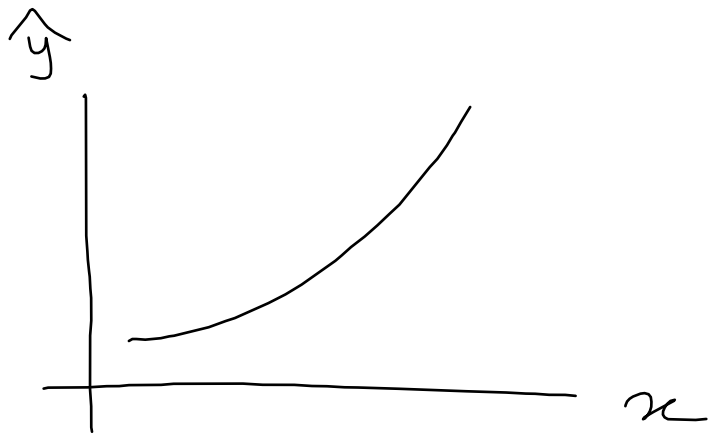
$$\hat{\beta}_1 < 0 \quad \& \quad \hat{\beta}_2 > 0$$



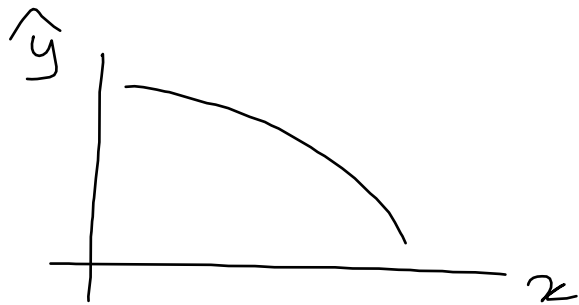
Turning pt. (min.) at

$$x^* = \left| \frac{\hat{\beta}_1}{2\hat{\beta}_2} \right|$$

$$\hat{\beta}_1 > 0 \quad \& \quad \hat{\beta}_2 > 0$$



$$\hat{\beta}_1 < 0 \quad \& \quad \hat{\beta}_2 < 0$$



Interaction Terms

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \cdot x_2 + u$$

$$\frac{\Delta y}{\Delta x_1} = \beta_1 + \beta_3 x_2$$

$$\frac{\Delta y}{\Delta x_2} = \beta_2 + \beta_3 x_1$$

e.g. $\frac{\Delta y}{\Delta x_1}$ at $x_2 = \bar{x}_2$: $\hat{\beta}_1 + \hat{\beta}_3 \bar{x}_2$

$$\text{bwght} = \beta_0 + \beta_1 \text{mage} + \beta_2 \text{npvis} + \beta_3 \text{mage} \cdot \text{npvis} + u$$

g mnp=mage*npvis

reg bwght mage npvis mnp

$$\hat{\beta}_1 = 28.65$$

$$\hat{\beta}_2 = 78.98$$

$$\hat{\beta}_3 = -2.2$$

. reg bwght mage npvis mnp

Source	SS	df	MS	Number of obs	=	1,764
Model	9077574.59	3	3025858.2	F(3, 1760)	=	9.11
Residual	584682222	1,760	332205.808	Prob > F	=	0.0000
				R-squared	=	0.0153
				Adj R-squared	=	0.0136
Total	593759796	1,763	336789.448	Root MSE	=	576.37

bwght	Coefficient	Std. err.	t	P> t	[95% conf. interval]	
mage	28.6513	9.424529	3.04	0.002	10.16685	47.13575
npvis	78.98256	22.92306	3.45	0.001	34.02327	123.9419
mnp	-2.200881	.7824839	-2.81	0.005	-3.735577	-.6661854
_cons	2396.552	273.9774	8.75	0.000	1859.197	2933.907

$$\frac{\Delta \text{bwght}}{\Delta \text{mage}} = \beta_1 + \beta_3 \text{npvis}$$

↓
avg. = 11.62

$$\text{at } \overline{\text{npvis}} = 28.65 + (-2.2) \times 11.62 = 3.086$$