Ch : 4 Inference

$\beta \longrightarrow$ unknown

test e.g. $\beta = 0$

using $\hat{\beta}$

3 ways of testing

- is $\hat{\beta}$ too far from hypothesized β
- how improbable is $\hat{\beta}$
- is hypoth. β in $\hat{\beta} - E , \hat{\beta} + E$

Sampling Distributions

$$\hat{\beta}_j = \beta_j + \text{a term linear in } u$$

its distribution follows from that of u

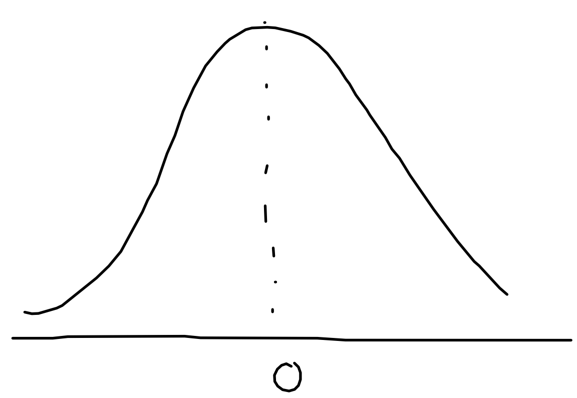
Assⁿ : u indep. of $x_1, \ldots, x_k$

normal

$$E(u) = 0$$

$$Var(u) = \sigma^2$$

$$u \sim N(0, \sigma^2)$$



Given all assⁿs up to normality

$$\hat{\beta}_j \sim N\left(\beta_j, var(\hat{\beta}_j)\right)$$

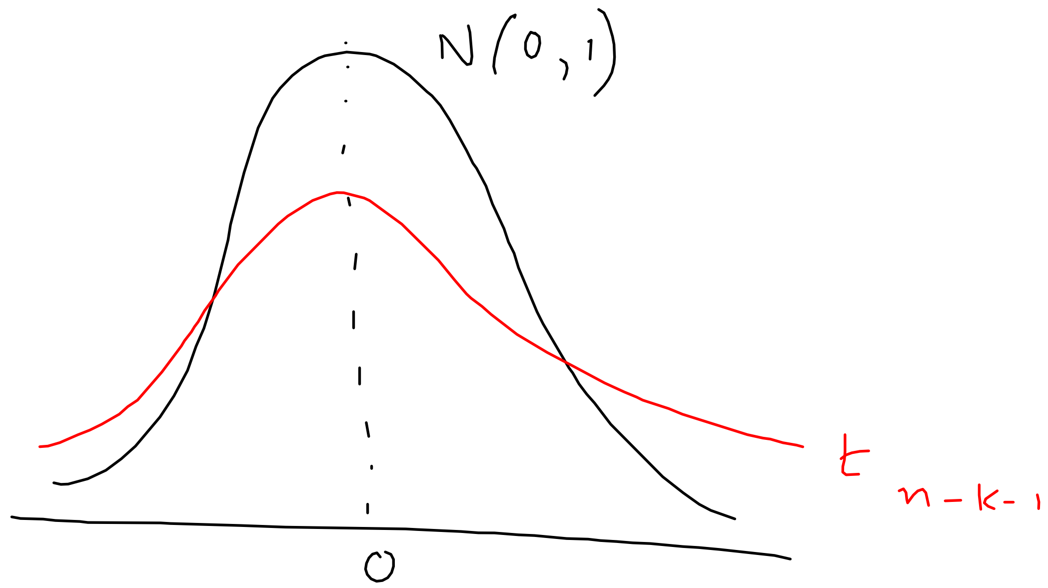
derived in Ch. 3

$$\frac{\hat{\beta}_j - \beta_j}{sd(\hat{\beta}_j)} \sim N(0, 1)$$

$$\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim t_{n-k-1}$$ (t dist. with deg. of freedom, $df = n-k-1$)

using $\hat{\sigma}$ in place of σ

$\rightarrow N(0,1)$ as $df \rightarrow \infty$



N(0,1)
t n-k-1
0

Single hypothesis – single parameter

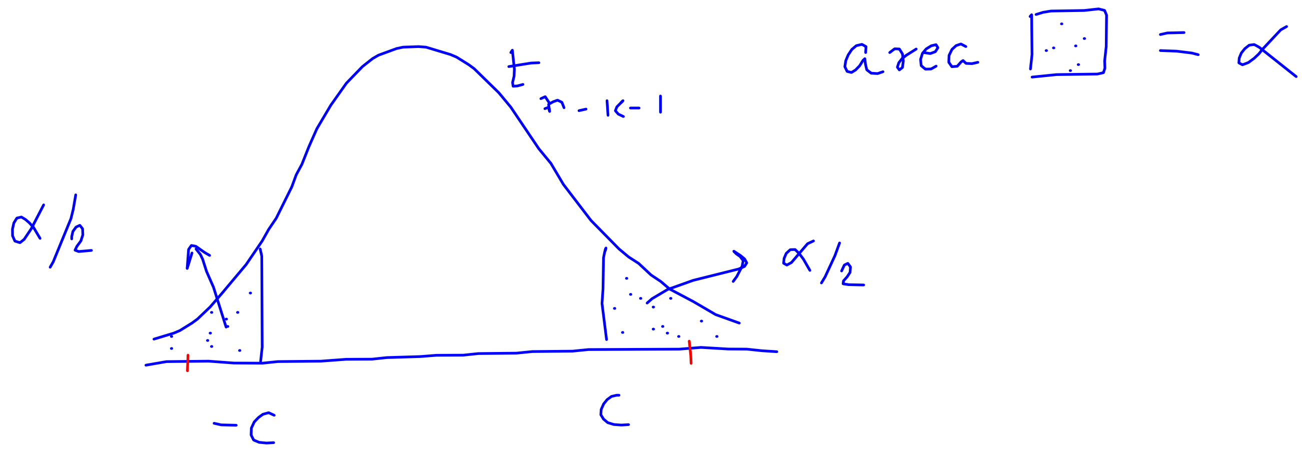
Null $H_0 : \beta_j = a_j$ $a_j = 0 \rightarrow$ special case

Alternative $H_1 : \beta_j \neq a_j$ $\rightarrow$ two-tailed test

$>$ or $<$: one-tailed test

Test statistic, or
t "
t ratio

$$t_{test} = \frac{\hat{\beta}_j - a_j}{se(\hat{\beta}_j)}$$



If H_0 is true, unlikely that $|t_{test}| > c$

Rejection rule: reject H_0 if $|t_{test}| > c$ else fail to reject H_0.

α = significance level (or size)

$= P(\text{rej. } H_0 \mid H_0 \text{ true})$

Equivalent decision rule: reject H_0 if a_j is beyond $c \cdot se(\hat{\beta}_j)$ from $\hat{\beta}_j$.

Fail to reject H_0 if a_j is within

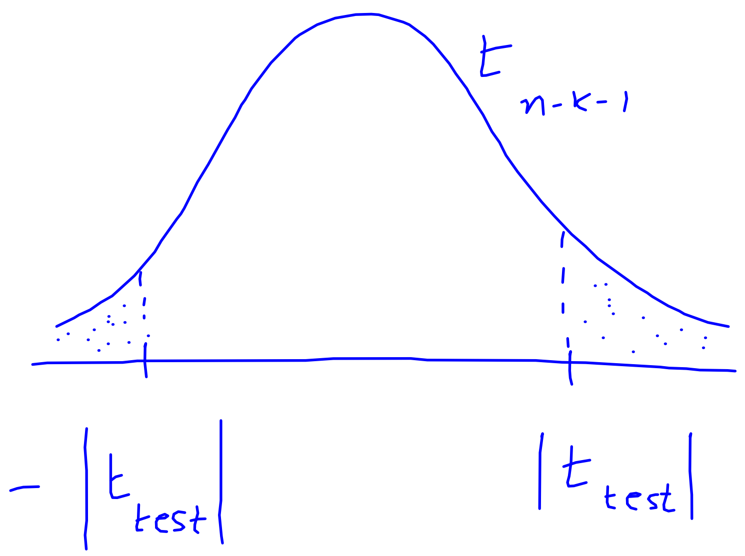
$$\left[\hat{\beta}_j - c \cdot se(\hat{\beta}_j),\ \hat{\beta}_j + c \cdot se(\hat{\beta}_j)\right]$$

$(1-\alpha)$ confidence interval for β_j: $\hat{\beta}_j \pm c \cdot se(\hat{\beta}_j)$

$\downarrow$

confidence level $= P(\text{not rej. } H_0 \mid H_0 \text{ true})$

Another equivalent rejection rule : reject H_0 if area beyond $|t_{test}|$ and $-|t_{test}|$ $< \alpha$.



area $\boxed{} = 2P(t > |t_{test}|)$
$\downarrow$
p-value

Rej. H_0 if p-value $< \alpha$.

NBASAL

$$wage = \beta_0 + \beta_1\, points + \beta_2\, rebounds + \beta_3\, assists + u$$

$$H_0 : \beta_1 = 0$$

$$H_1 : \beta_1 \neq 0$$

$$t_{test} = \frac{\hat{\beta}_1}{se(\hat{\beta}_1)} = 7.02 = \frac{81.19}{11.569}$$

c for t_{n-k-1}

$n-k-1$

$= 269 - 3 - 1$

$= 265$

$\alpha = 0.05$

$c = 1.96$

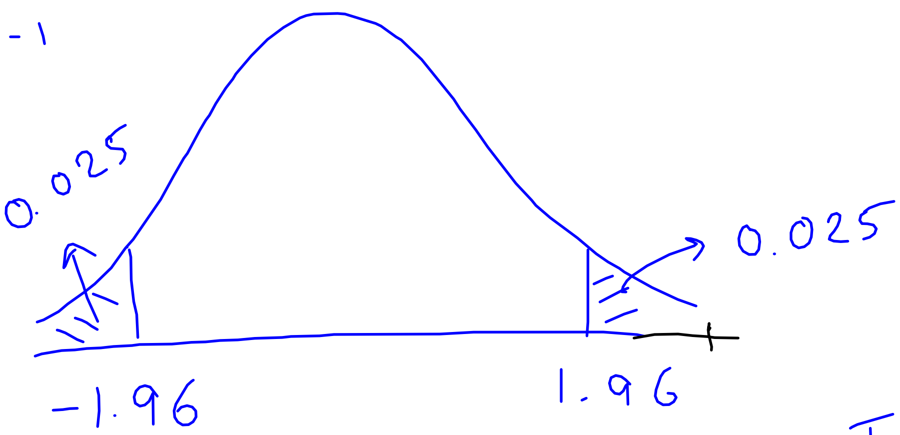


Table G.2 : t

Table G.1 : $N(0,1)$ or Z

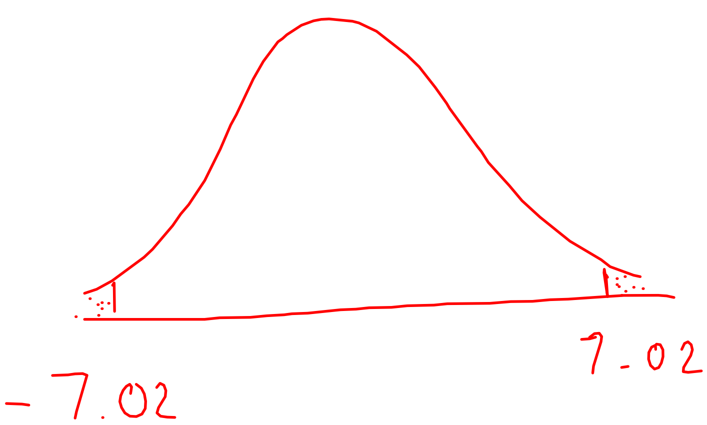
Rej. H_0 $\because$ $t_{test} > 1.96$

0.95 CI : $\hat{\beta}_1 \pm c \cdot se(\hat{\beta}_1)$

$\hat{\beta}_1 = 81.19$, $c = 1.96$, $se(\hat{\beta}_1) = 11.569$

$[58.41, 103.97]$

Rej. H_0 $\because$ CI excludes 0.



p-value practically zero.

Rej. H_0 $\because$ p-value < 0.05.

$H_0 : \beta_1 = 70$

$H_1 : \beta_1 \neq 70$

$$t_{test} = \frac{\hat{\beta}_1 - 70}{se(\hat{\beta}_1)} = 0.967$$

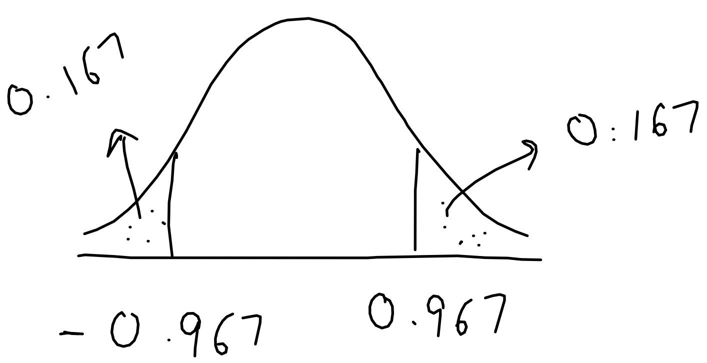
$\rightarrow |t_{test}| < 1.96$

Fail to rej. H_0 $\longrightarrow$ 70 is in CI

$\rightarrow$ p-value > 0.05

$= 2 \times 0.167$

$= 0.334$



$a_j = 0 \rightarrow$ test of statistical significance of x_j

$\hat{\beta}_j$: economic significance

t_{test} : statistical "

Multiple Hypotheses

$H_0 : \beta_j = 0, \beta_\ell = 0$

H_1 : at least β_j or $\beta_\ell \neq 0$

unrestricted model : H_0 not imposed

restricted model : H_0 imposed by omitting x_j and x_ℓ

Test statistic : based on comparing fit across the 2 models

Follows F distribution.

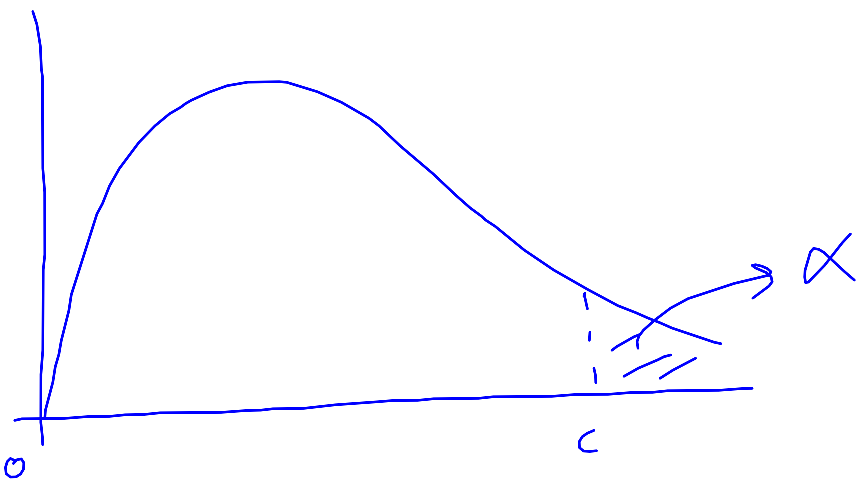
$$F_{test} = \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)}$$

q : numerator df (# β's tested)

$n-k-1$: denominator df

$F_{test} \sim F_{q,\, n-k-1}$

$t^2_{n-k-1} = F_{1,\, n-k-1}$



Reject H_0 if $F_{test} > c$

critical values: Tables G.3a, G.3b, & G.3c

Also, $F_{test} = \dfrac{(R^2_{ur} - R^2_{r})/q}{(1 - R^2_{ur})/(n-k-1)}$

NBASAL:

$$wage = \beta_0 + \beta_1\,points + \beta_2\,reb. + \beta_3\,assists + u$$

$H_0: \beta_1 = 0,\ \beta_2 = 0$

$H_1:$ not H_0

$$F_{test} = 93.52$$

c for $F_{2,265} = 3.03$ (using Stata or Table G.3b)

$\alpha = 0.05$

Reject H_0.