

## Ch. 3 (Mult. lin. reg.) contd.

$k$  : # of indep vars.

e.g.  $k = 3$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

salary

pts.

rebounds

assists

prices

sq.ft.

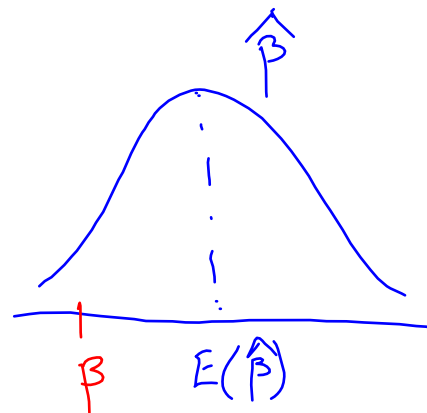
bdrms.

lot size

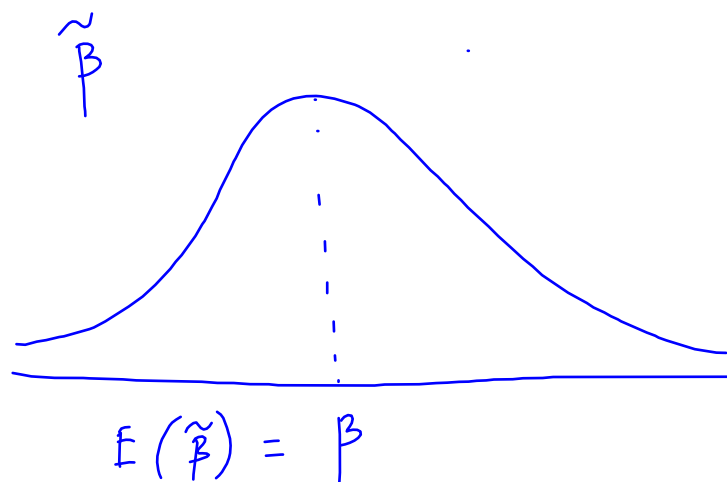
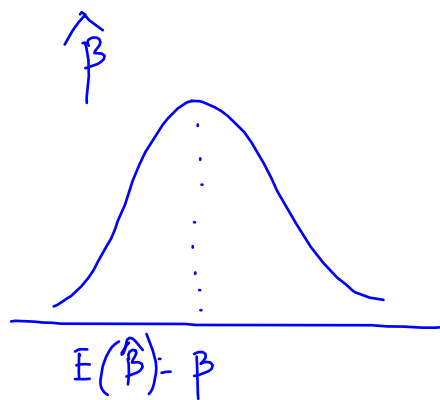
omitted variable  $\rightarrow$  arising from omitting  
bias a var. that affects  $y$   
and is correlated with  
indep. vars.

e.g.  $u \rightarrow$  games / yr.  
 $\rightarrow$  quality

$$E(\hat{\beta}) \neq \beta$$



## Variance



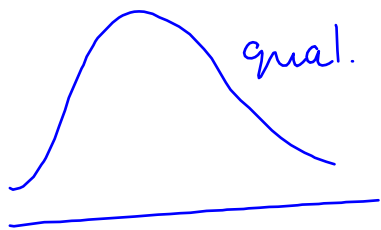
Both unbiased  
but  $\text{var}(\hat{\beta}) < \text{var}(\tilde{\beta})$

Homoskedasticity :  $\text{var}(u | x_1, x_2, x_3) = \sigma^2$

$$x_1 = 2000$$

$$x_2 = 4$$

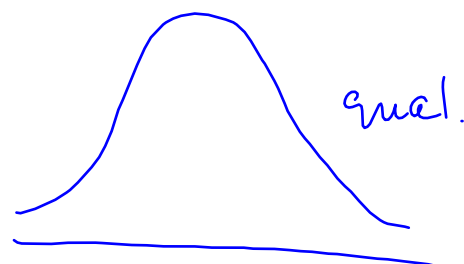
$$x_3 = 1$$



$$x_1 = 1000$$

$$x_2 = 1$$

$$x_3 = 0.5$$



$$\text{var}(\hat{\beta}_j) = \frac{\sigma^2}{\text{SST}_j (1 - R_j^2)} \quad j=1, 2, 3$$

e.g.  $y$ : price  
 $x_1$ : sq.ft.  
 $x_2$ : # bdrm.  
 $x_3$ : lot size

$$\text{var}(\hat{\beta}_1) = \frac{\sigma^2}{\text{SST}_1 (1 - R_1^2)}$$

$\swarrow$  Total variation in  $x_1$ , or sq.ft.  
 $\searrow$   $R^2$  from reg. of  $x_1$  on  $x_2$  &  $x_3$   
 sq.ft.      bdrms      lot size

$$\sum_{i=1}^n (x_{i1} - \bar{x}_1)^2$$

$i=1, \dots, n$

$R_j^2$  close to 1  $\rightarrow$  multicollinearity  
 does not violate ass<sup>n</sup>s  
 req'd. for unbiasedness

Under ass<sup>n</sup>s up to homoskedasticity

$\hat{\sigma}^2 \rightarrow$  unbiased estimator of  $\sigma^2$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^n \hat{u}_i^2}{n-k-1}$$

k: # of indep.  
vars.

$$se(\hat{\beta}_j) = \frac{\hat{\sigma}}{\sqrt{SST_j(1-R_j^2)}}$$

$j = 1, 2, 3$