

Suppose we have two independent samples drawn from independent populations. While the sample average values are  $\bar{x}_1$  and  $\bar{x}_2$ , the corresponding population means are  $\mu_1$  and  $\mu_2$ . Also, suppose the null hypothesis is  $H_0: \mu_1 - \mu_2 = 0$ . In this case, the test statistic is given by  $\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{SD}$ , where SD denotes the standard deviation of  $\bar{x}_1 - \bar{x}_2$ .

Suppose we are given three scenarios:

1. The population standard deviations are  $\sigma_1$  and  $\sigma_2$  and known.
2. The population standard deviations are unknown but assumed equal.
3. The population standard deviations are unknown and not assumed equal.

The possible values of SD are

- $\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}$  with the pooled variance  $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$
- $\sqrt{\left( \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)}$
- $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Which of the above SD expressions correspond to Scenarios 1, 2, and 3?

The possible test statistics follow a

- $t$  distribution with  $n_1 + n_2 - 2$  degrees of freedom
- $z$  distribution
- $t$  distribution with degrees of freedom given by  $\min\{n_1 - 1, n_2 - 1\}$

Which of the above test statistics correspond to Scenarios 1, 2, and 3?